Compressive strength optimisation of rice husk ash concrete using Scheffe's mathematical model

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Abstract

The high cost of cement as a significant component of concrete has led to the high cost of concrete production in most developing countries. Because of its longevity and good benefit-to-cost ratio, blended cement has grown in popularity in developed countries. Rice husk ash (RHA) is a residue produced by the burning of rice husk that is abundant in rice mills. RHA has been proven as a good supplementary cementitious material for concrete production due to its low energy requirements, minimal greenhouse gas emissions during processing and service life, and strong pozzolanic reaction. Using Scheffe's (4, 2) simplex-lattice design, a mathematical model was developed to optimise the compressive strength of RHA reinforced concrete in this research. RHA was used as the second component in concrete, along with water, cement, fine and coarse aggregates, at a partial replacement ratio of 20% in cement. The compressive strength of RHA concrete was determined using Scheffe's Simplex technique for the various componential ratios as well as the control points that would be used to validate the Scheffe's model. The model's adequacy was assessed using the f-statistics test, the student's t-test, and ANOVA at a 5% significance level. The statistical result shows a satisfactory correlation between the values produced from the developed Scheffe's model and the control laboratory data. The maximum compressive strength of RHA concrete obtained was 40.75 N/mm^2 corresponding to a mix ratio of 0.475: 1.0: 2.75: 3.50 and the minimum compressive strength obtained was 7.41 N/mm^2 corresponding to a mix ratio of 0.47: 1.0: 2.5: 4.5 for water, binder (80% cement and 20% RHA), fine aggregate, and coarse aggregate, respectively. The ratio of the mix elements to a particular required compressive strength value may be calculated with a high degree of precision using the established Scheffe's simplex model, while also giving the answer in less time by resolving trial mix challenges.

Keywords: compressive strength, concrete, optimisation, Scheffe's model, rice husk ash
Kulcsszavak: nyomószilárdság, beton, optimalizálás, Scheffe-modell, rizshéj pernye

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1. Introduction

Concrete compressive strength is one of the most significant material properties utilised in structural design and quality control [1]. Concrete is mainly utilised in civil engineering under compression loading situations because its compressive strength is significantly greater than its tensile and/or flexural strengths. Furthermore, because it is directly connected to the structure of the hydrated cement paste, compressive strength is frequently regarded as a diagnostic of concrete quality [2]. For these reasons, compressive strength is commonly used to make choices about the strength and serviceability of concrete components and structures [3]. The compressive strength of concrete is one of its major engineering features, and it has become a standard in industrial practice to classify concrete based on its compressive strength (i.e. grades) [4]. The compressive strength of concrete is defined by its capacity to withstand cracking and fissure failure. The ultimate compressive strength of a material is equal to the value of uniaxial compression stress attained at the complete failure point and is affected by factors such as constituent compressive strength, water-cement ratio, material quality, curing methods, air entrainment, temperature effects, and mixture constituent proportion. Concrete compressive strength is proportional to its density. Additionally, it is dependent on the mix proportion, aggregates, cement properties, water-cement ratio, curing duration, and SCMs replacement level [5-6]. The experimental technique of concrete mix design consists of a series of lengthy experiments that are mostly based on trial and error and entail approximate estimations based on practical experience without the use of a mathematical or statistical scientific approach [7]. To reduce the number of experimentation tests required before determining the ideal combination ratio for concrete mixed with RHA, an analytical approach that tries to justify the initial trial mix into a logical and systematic procedure is being developed. Based on established knowledge of certain empirical connections, particular weights of mixture ingredients, and findings from previous literatures, this will aid in determining the optimal combination for the mixture ingredients in spending less resources [8].

Scheffe's approach is a mixed model strategy for adjusting statistical significance levels in a linear regression study to account for multiple comparisons. When performing evaluation of simultaneous confidence levels for regression analysis using objective functions [9], it is critical for a particular kind of regression analysis known as analysis of variance. Scheffe's simplex second order regression model is created statistically to maximise the compressive strength attribute of RHA concrete. The statistical method used for recycling, use, and re-use of agricultural solid waste material such as RHA has been proven to be a beneficial strategy in engineering practice [4-10]. The application of Scheffe's simplex lattice design to accomplish mixture design has been used in various civil engineering applications to provide solutions in fields such as material science, pavement material changes, soil stabilisation, geotechnical and concrete technology [11-13]. Ambrose et al. [14]; in their work on Compressive strength and Scheffe's optimisation of mechanical properties of recycled ceramics tile aggregate concrete. Laboratory tests were performed in relation to the calculated Scheffe's design point. To validate the established mathematical model, statistical analysis was performed. In their results, the maximum predictable response from the compressive strength model was 42.13 N/mm^2 existing at vertex X_5 of the simplex and corresponding to mix ratio of 0.45:1:0:1:2 for water, cement, sand, recycled-ceramic tiles (CRT) and coarse aggregates (CA). Conversely, the lowest predictable compressive strength was found to be 20.34 N/mm^2 existing very close to Vertex X_3 and corresponding to mix ratio of 0.64:1:2.35:0.06:4.35. Their investigation also revealed that CRT, which is widely available and inexpensive, has been effectively employed to produce concrete. Also, Alaneme and Mbadike [15]; in their research study on compressive strength modelling of palm nut fiber concrete using Scheffe's theory. The concrete mixture consists of five components: cement, water, coarse aggregates, fine aggregates, and palm-nut fiber, an agricultural waste. At a strength value of 31.53 N/mm^2, the best combination ratio of 0.525:1.0:1.45:1.75:0.6 was found, while the lowest combination ratio of 0.6:1.0:1.8:2.5:1.2 for water, cement, fine and coarse aggregate, and palm nut fiber was reached at a strength value of 17.235 N/mm^2. Furthermore, Chiemela et al. [45] used Scheffe's theory to model the compressive strength property of concrete when given componential ratios, as well as predict the corresponding portions of the mixture ingredients with prescribed values of compressive strength vale of concrete obtained by substituting quarry dust for river sand in their work. The formulated model was then put to the test with the response value of the control point. F-statistics and a student's t-test with a 95% confidence level were employed in this statistical investigation. The results demonstrate that the anticipated and measured values are not significantly different.

There are currently no mathematical models for RHA concrete, but the demand for such models is essential. Therefore, this research seeks to investigate the use of RHA, a residual agricultural waste, as a partial replacement in the second component (binder) in concrete mixtures. The goal of this study is to partially integrate ash material at 20% replacement level into concrete mixtures rather than using it as a separate component, and to use statistical methods to determine the best mixture combination of the concrete mixture's ingredients, which include water, cement, RHA, fine aggregate, and coarse aggregate. RHA is being employed in this study to improve ecological infrastructure development by recycling solid wastes originating from agricultural or industrial operations, which is accomplished by substituting traditional concrete materials. The compressive strength of concrete is typically influenced by the mix proportions of its constituents. The optimal combination ratio for the mix elements of water, binder (cement and RHA) fine aggregate, and coarse aggregate was determined using Scheffe's optimisation approach to predict the concrete compressive strength behaviour. This study will add to the body of information regarding the optimisation of concrete mixtures including solid waste materials as an additive. The findings of this study will allow for improved choice in terms of determining concrete grade and batching of its constituents for structural application. Additionally, this will simplify mix designs by resolving trial mix issues, reducing experimental blunders.

1.1 The Scheffe's Simplex-lattice Design

Simplex lattice design is a type of mixed experiment that is used to study response and component correlations. Simplex lattice designs are simply referred to as Scheffe's simplex lattice designs in Scheffe's theory. Mixture experiment approaches are generally used in instances when the response is determined by the mass or volume proportions of individual components rather than their overall mass or volume, as is common of concrete properties [9, 17]. According to [4], if q indicates the number of mixture components, X_1, X_2, X_3, X_4....,Xq, however, y signifies the intended response. No component has a negative value in a mixing experiment, and the sum of the component ratios must be one.

The simplex-lattice is an orderly arrangement of lines that connects the expected experimental points of the mixed constituent ratio design. The factor space in a q-component mixing experiment is a regular {q-1} simplex [18]. If q = 2, the lattice simplex is a straight line; if q = 3, it is an equilateral triangle; and if q = 4, it is a regular tetrahedron with each vertex representing one of the components. However, Scheffe proposed that in a mixture design, each component of the mixture resides on a vertex of a simplex lattice with {q-1} factor space, such that if the degree of the polynomial to be fitted to the design is denoted by n, then a {q, n} simplex lattice for q-components consists of uniformly spaced points defined by all the possible combinations of {n+1} levels of each component [19]. Concrete's properties are determined by the appropriate mass or volume mix proportion of its ingredients, not by its overall mass or volume. Scheffe's optimisation theory, as a result, may be employed to model and optimise concrete properties. According to [14], for a 4-compomponent mixture adopted in this work, the reduced second-degree polynomial can be obtained as follows:

$$\sum X_i = 1 \text{ or } X_1 + X_2 + X_3 + X_4 = 1 \qquad (1)$$

Where; X_1= Water/Cement Ratio; X_2 = Binder (80% OPC and 20% RHA); X_3 = Fine Aggregates (Sand); X_4 = Coarse Aggregates (Granite).

$$Y = \propto_i X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_{12} X_1X_2 + \propto_{13} X_1X_3 + \propto_{14} X_1X_4 + \propto_{23} X_2X_3 + \propto_{24} X_2X_4 + \propto_{34} X_3X_4 \quad (2)$$

The number of terms in the simplified polynomial is also related to the design points on Scheffe's lattice simplex. As a result, the equations' coefficients can be expressed as functions of expected responses (y_i) at the simplex's design and control points. The general relationship between the two is as follows:

$$a_i = y_i \quad (3)$$

And for a (4,2) polynomial

$$a_{ij} = y_i = 4_{ij} - 2y_i - 2y_j \quad (4)$$

Obam [20] also considered experiments with mixtures in which the property he studied was determined by the proportions of the components but not by their quantities in the mixture. The relationship between the compressive strength of concrete and the proportion of w/c (water/cement), cement, fine and coarse aggregates is an obvious example of such a study. If a mixture has q components in total, and Xi is the proportion of the components (ingredients) of the ith component in the mixture such that

Xi ≥ 0 (i =1-4) (2) (5)

Then, assuming the mixture to be a unit quantity, he calculated that the sum of all the proportions of the components must equal unity. That is to say

$$\sum_{i=1}^{4} X_i = 1 \quad (6)$$

where in this case,
X_1 is proportion of water/cement (w/c) ratio
X_2 is proportion of cement
X_3 is proportion of sand
X_4 is proportion of crushed stone.
Thus, the factor space is a regular (q-l) dimensional simplex.

1.1.1 Components relationship in Scheffe's factor space

For a quaternary system, q = 4, the regular simplex is a tetrahedron where each vertex represents a straight component, an edge represents a binary system and a face a tertiary one. Points inside the tetrahedron correspond to quaternary systems as shown below. Each point in the tetrahedron therefore represents a certain composition of the quaternary system. The component X_1 is therefore absent in the face X_2, X_3, X_4 but as tetrahedron sections parallel to the face approach vertex X_1, component X_1 in them grows concentration.

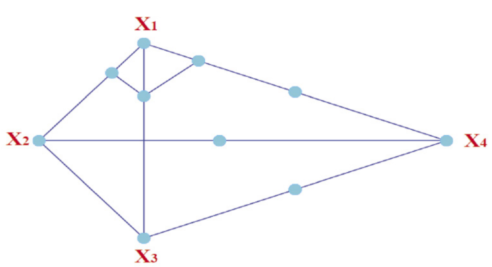

Fig. 1 A {4.2} Scheffe's simplex lattice with the tetrahedron and corresponding points
1. ábra A {4.2} Scheffe tetraéder szimplex rács a hozzátartozó csomópontokkal

The mixed components are uniformly dispersed in Scheffe's simplex design, and the proportions assumed by each component are n+1 equally spaced levels from 0 to 1 according to Eq. (7).

$$X_i = 0, \frac{1}{n}, \frac{2}{n}, \ldots..1 \quad (7)$$

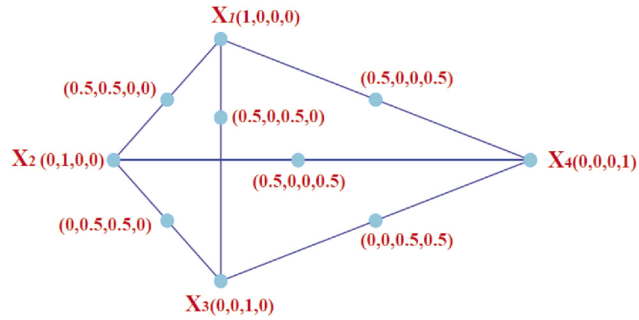

Fig. 2 A {4.2} Scheffe's simplex lattice with pseudo ratios at design points
2. ábra A {4.2} Scheffe szimplex rács pszeudoarányokkal a tervezési pontokban

The factor space for a 4,2 Scheffe's simplex lattice, as illustrated in *Fig. 2* above, is a tetrahedron, and each component has the proportions 0, and 1. There are 10 points at the tetrahedron's borders and vertices, which correspond to the number of terms in the simplified second-degree polynomial in Eq. (4). The four points indicated by (1,0,0,0); (0,1,0,0); (0,0,1,0); and (0,0,0,1) at the vertices represent single component mixes, whereas the remaining six points in the middle of each edge represent binary blends of two component mixtures [14].

2. Methodology

The investigation in this work was divided into two stages. The compressive strength of concrete with varied degrees of RHA substitution with Portland limestone cement (PLC) was investigated in the first stage (part A). Polynomial models were developed in the second stage (part B) for optimisation and prediction of compressive strength of RHA concrete using Scheffe's simplex lattice model.

2.1 Laboratory experiment

Water, cement, RHA, fine aggregates (river sand), and coarse aggregates (granite chippings) were used in laboratory tests in both stages of this study. The cement utilised in this study, Unicem brand of Portland Limestone cement of grade 32.5R was used and it met the requirements of the CEM II class of cements as described in NIS 444-1 [21]. The river sand was collected from a river sand mining location in Nsukka, Enugu State, while the granite chippings were sourced from a quarry in Abakaliki, Ebonyi State, all in Nigeria. The rice husks were obtained from Ogoja in Cross River State. They were burned in the open air, and the ash was collected and stored in a dry place in the laboratory. Physical examination revealed that RHA obtained was greyish in colour after being burned. The ashes were chemically analysed to identify the elemental content of each ash.

The main elemental oxide composition of rice husk ash (RHA) was determined using X-Ray Fluorescence at the Standard Organisation of Nigeria (SON) Engineering laboratory, Enugu State Office, Emene Industrial Layout, Enugu State, Nigeria. The compressive strength test is used to determine the behavior of materials under compression. Compressive strength is widely regarded as the most significant feature of concrete. Three duplicate concrete samples were produced in 150 mm × 150 mm × 150 mm moulds for each mix ratio. Following the mixing and casting of the concrete, the specimens were removed from the mould and cured for 28 days in a curing tank before being tested for compressive strength in accordance with BS EN 12390 [22]. The compressive strength of concrete was calculated using the formula:

$$\sigma = \frac{P}{A} \tag{8}$$

where P is the failure load; A is the cross sectional area of the concrete cube.

2.2 Mathematical modelling

The number of design components in the second phase of this study, which required modelling, was four, and the data would be fitted into Scheffe's second-degree polynomial. As a result, the mixed experiment was planned with a 4, 2 simplex lattice using a commercial statistical software and the design matrix is shown in *Tables 1* and *2*. The simplex condition that $X_1+X_2+X_3+X_4=1$ makes it impossible to use standard mix ratios such as 1:3:6 at a given water-cement ratio. As a result, a modification of the real components (normal mix ratios) is required to fulfill this criterion. The design matrix for the Xi experimental points given in *Table 1* is referred to as "Peudo-components," whereas Zi are the actual experimental components.

$$X=AZ \tag{9}$$

Where A is the inverse of Z matrix and

$$Z=AX^T \tag{10}$$

Where A is the inverse of Z matrix, X^T is the transpose of matrix X.

Table 1 presents the values of the computed real components (Z_1, Z_2, Z_3, Z_4), whereas *Table 2* shows the values for the control locations.

S/N	X1	X2	X3	X4	Res-ponse	Z1 Water	Z2 Binder	Z3 FA	Z4 CA
1.	1	0	0	0	Y_1	0.45	0.50	0.46	0.44
2.	0	1	0	0	Y_2	1	1	1	1
3.	0	0	1	0	Y_3	1.5	2.0	2.5	3.0
4.	0	0	0	1	Y_4	3	4.0	5.0	6.0
5.	½	½	0	0	Y_{12}	0.475	1	2.75	3.5
6.	½	0	½	0	Y_{13}	0.455	1	2.0	5.0
7.	½	0	0	½	Y_{14}	0.445	1	2.25	4.5
8.	0	½	½	0	Y_{23}	0.48	1	2.25	4.5
9.	0	½	0	½	Y_{24}	0.47	1	2.5	4.5
10.	0	0	½	½	Y_{34}	0.45	1	2.75	5.5

Binder (PLC-80%, RHA-20%); FA-fine aggregate; CA-coarse aggregate

Table 1 Actual and pseudo components for Scheffe's {4, 2} simplex lattice
1. táblázat Scheffe {4, 2} szimplex rácsának aktuális és pszeudo komponensei

S/N	X1	X2	X3	X4	Res-ponse	Z1 Water	Z2 Binder	Z3 FA	Z4 CA
11.	½	¼	1/4	0	C_1	0.465	1	1.88	3.75
12.	¼	¼	1/4	1/4	C_2	0.463	1	2.25	4.5
13.	0	¼	0	3/4	C_3	0.46	1	2.63	5.5
14.	½	0	1/4	1/4	C_4	0.48	1	2.13	4.25
15.	½	¼	0	1/4	C_5	0.46	1	2.0	4.0
16.	0	¼	3/4	0	C_6	0.47	1	2.38	4.75
17.	0	½	1/4	1/4	C_7	0.475	1	2.13	4.75
18.	¼	1/8	1/2	1/8	C_8	0.46	1	2.25	4.50
19.	¼	¼	0	½	C_9	0.458	1	2.38	4.75
20.	1/8	1/8	1/4	½	C_{10}	0.454	1	2.56	5.13

Table 2 Control points for Scheffe's {4, 2} simplex lattice
2. táblázat Scheffe {4, 2} szimplex rácsának kontrolpontjai

S/N	Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
1	0.08	0.23	0.23	0.46	0.018	0.018496	0.036993	0.052847	0.105694	0.105694
2	0.07	0.168	0.253	0.51	0.012	0.018706	0.037411	0.042513	0.085025	0.127538
3	0.07	0.155	0.31	0.47	0.011	0.021633	0.03245	0.048074	0.072111	0.144222
4	0.05	0.095	0.286	0.57	0.005	0.013605	0.027211	0.027211	0.054422	0.163265
5	0.06	0.135	0.269	0.54	0.008	0.015578	0.031157	0.036229	0.072457	0.144915
6	0.05	0.111	0.278	0.56	0.006	0.014881	0.029762	0.031002	0.062004	0.155009
7	0.04	0.087	0.348	0.52	0.004	0.015399	0.023098	0.030193	0.04529	0.181159
8	0.04	0.107	0.322	0.54	0.004	0.011373	0.018955	0.034463	0.057439	0.172317
9	0.06	0.117	0.234	0.58	0.008	0.015047	0.037618	0.027359	0.068397	0.136794
10	0.06	0.099	0.248	0.59	0.006	0.014704	0.035291	0.024507	0.058818	0.147044

Table 3 Z^T Matrix based on Eq. (14)
3. táblázat A Z^T matrix a 14-es egyenlet alapján

α_1 = 35.72, α_2 = 27.99, α_3 = 23.1, α_4 = 18.23

From *Eq. (4)*

$$\propto_{12} = 4\,(40.75) - 2(35.72) - 2(27.99) = 35.58$$
$$\propto_{13} = 4(34.6) - 2(35.72) - 2(23.1) = 20.76$$
$$\propto_{14} = 4(17.35) - 2(35) - 2(24) = -48.6$$
$$\propto_{23} = 4(30.6) - 2(27.99) - 2(23.1) = 20.22$$
$$\propto_{24} = 4(7.42) - 2(27.99) - 2(18.23) = -62.76$$
$$\propto_{34} = 4(24.63) - 2(23.1) - 2(18.23) = 15.86$$

Thus, from *Eq (2)*, we have;

$$\sigma_c = 35.7X_1 + 27.99X_2 + 23.1X_3 + 18.23X_4 + 35.42X_1X_2 + 20.56X_1X_3 - 47.60X_1X_4 + 20.22X_2X_3 - 62.76X_2X_4 + 15.86X_3X_4 \tag{11}$$

Eq. (11) is the mathematical model for the optimization of the compressive strength of RHA concrete based on Sheffe's (4,2) polynomial.

3. Results and discussion

3.1 Material characterisation

Fig. 3 shows the results of the particle size distribution study performed on cement and non-ground RHA. The results reveal that the percentage passage of non-ground RHA is equivalent to cement and, as such, should be expected to play a pozzolanic role as well as a microfiller effect in order to increase the particle packing density of concrete. The chemical composition of the ash used for the experiment is shown in *Table 4* below; the findings shows that RHA is a highly reactive pozzolana due to a combined SiO_2, and Fe_2O_3 content of 85.60 percent, which is greater than the minimum value of 70 percent prescribed in ASTM C 618 [23].

Fig. 4 depicts the compressive strength of concrete at various levels of RHA replacement. As a consequence, it is clear that incorporating RHA enhances the compressive strength of the resultant concrete substantially.

S/N	Samples	Chemical composition (%)							
		ZnO	SiO_2	CaO	Fe_2O_3	K_2O	MnO	MgO	Na_2O
1	Cement (PLC)	0.12	23.5	65.2	3.40	0.40	0.18	1.35	0.30
2	RHA	0.75	84.6	0.30	0.25	0.69	0.43	0.45	0.51

Table 4 Chemical composition of PLC and RHA
4. táblázat A PLC és az RHA kémiai összetétele

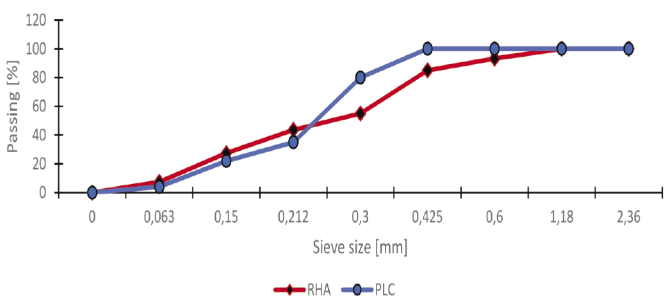

Fig. 3 Particle size distribution for RHA and PLC
3. ábra RHA és PLC szemcseméret-eloszlása

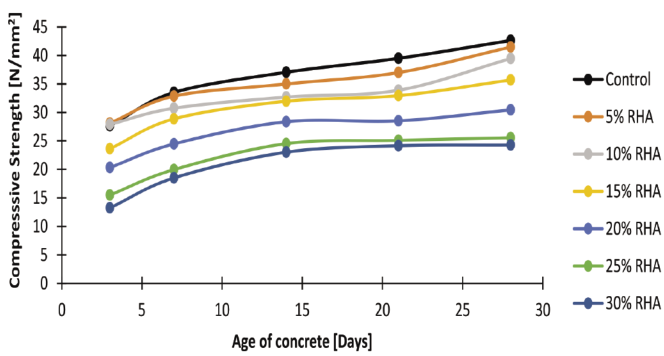

Fig. 4 Compressive strength result at different RHA replacement levels
4. ábra Nyomószilárdság eredménye különböző RHA helyettesítési szinteken

3.2 Mathematical modelling results

S/N	X_1	X_2	X_3	X_4	Lab. response	Z_1 Water	Z_2 Binder	Z_3 FA	Z_4 CA
1.	1	0	0	0	35.72	0.45	0.50	0.46	0.44
2.	0	1	0	0	27.99	1	1	1	1
3.	0	0	1	0	23.10	1.5	2.0	2.5	3.0
4.	0	0	0	1	18.23	3	4.0	5.0	6.0
5.	½	½	0	0	40.75	0.475	1	2.75	3.5
6.	½	0	½	0	34.64	0.455	1	2.0	5.0
7.	½	0	0	½	17.50	0.445	1	2.25	4.5
8.	0	½	½	0	30.67	0.48	1	2.25	4.5
9.	0	½	0	½	7.41	0.47	1	2.5	4.5
10.	0	0	½	½	24.63	0.45	1	2.75	5.5

Table 5 Compressive strength test results and replication based on Scheffe's (4,2) simplex lattice
5. táblázat Nyomószilárdság-vizsgálati eredmények és azok visszabontása a Scheffe-féle (4,2) szimplex rács alapján

S/N	X_1	X_2	X_3	X_4	Response	Z_1 Water	Z_2 Binder	Z_3 FA	Z_4 CA
11.	½	¼	1/4	0	31.09	0.465	1	1.88	3.75
12.	¼	¼	1/4	1/4	37.72	0.463	1	2.25	4.5
13.	0	¼	0	3/4	19.37	0.46	1	2.63	5.5
14.	½	0	1/4	1/4	31.22	0.48	1	2.13	4.25
15.	½	¼	0	1/4	20.25	0.46	1	2.0	4.0
16.	0	¼	3/4	0	24.23	0.47	1	2.38	4.75
17.	0	½	1/4	1/4	26.09	0.475	1	2.13	4.75
18.	¼	1/8	1/2	1/8	37.33	0.46	1	2.25	4.50
19.	¼	¼	0	½	31.67	0.458	1	2.38	4.75
20.	1/8	1/8	1/4	½	36.67	0.454	1	2.56	5.13

Table 6 Control points
6. táblázat Kontrol pontok

Response Symbol	Lab. Response (Y_k)	Model Response (Y_E)	$Y_K - \overline{Y_K}$	$Y_E - \overline{Y_E}$	$(Y_K - \overline{Y_K})^2$	$(Y_E - \overline{Y_E})^2$
C1	31.09	29.57	1.525	-0.654	2.325625	0.427716
C2	37.72	37.72	8.155	7.496	66.50402	56.19002
C3	19.37	19.57	-10.195	-10.654	103.938	113.5077
C4	31.22	30.41	1.655	0.186	2.739025	0.034596
C5	20.25	23.52	-9.315	-6.704	86.76923	44.94362
C6	24.23	24.26	-5.335	-5.964	28.46223	35.5693
C7	26.1	26.10	-3.465	-4.124	12.00623	17.00738
C8	37.33	39.24	7.765	9.016	60.29522	81.28826
C9	31.67	33.20	2.105	2.976	4.431025	8.856576
C10	36.67	38.65	7.105	8.426	50.48103	70.99748
$\sum$	295.7	302.24			417.9517	428.8226
Mean	29.57	30.224			46.43907	47.64696
	F Critical one-tail=	3.178893		F =	1.02601	

Table 7 F-statistics test on experimental and model results for the control points
7. táblázat A kontrollpontok kísérleti és modelleredményeinek F-statisztikai vizsgálata

	Laboratory Response	Model Response
Mean	29.565	30.224
Variance	46.43907222	47.64696
Observations	10	10
Pearson Correlation	0.977327267	
Hypothesized Mean Difference	0	
Df	9	
t Stat	-1.424297987	
P(T<=t) one-tail	0.0940481	
t Critical one-tail	1.833112933	
P(T<=t) two-tail	0.188096201	
t Critical two-tail	2.262157163	

Table 8 T-Statistical tests on experimental and model results
8. táblázat A kísérleti és modelleredmények T-statisztikai vizsgálata

Group	Count	Sum	Average	Variance
Laboratory response	10	295.65	29.565	46.43907
Model response	10	302.24	30.224	47.64696

Table 9 ANOVA: Single Factor
9. táblázat Egyfaktoros ANOVA

3.3 Model validation and test for adequacy

To ascertain whether the formulated model in Eq. (11) is acceptable to be used in predicting compressive strength, it is essential to carry out statistical test. The test for adequacy of the model was carried out with the aid of f-statistics test, student's t test and analysis of variance. Fifteen extra points were used to test the model's validity and adequacy of the model was tested by comparing the experimental results of the control points with the predicted results. In this test, the two hypotheses tested are that: There is no significant difference between the obtained laboratory results of the compressive strength and the model predicted values at 0.05 critical value (α), this is the null hypothesis. There is a significant difference between the obtained laboratory results of the compressive strength and the model predicted values at 0.05 critical value (α), this is the alternate hypothesis. The F-test two-sample for variance was used to compare the two laboratory and model results. If $F > F_{crit}$, we reject the null hypothesis. *Table 7* shows the analytical results: $F = 1.02601$ and $F_{crit} = 3.178893$, indicating that $F_{crit} > F$. As a result, we do not reject the null hypothesis. This, however, implies that there was no substantial difference between the experiment and model results. As a result, the model is now suitable for predicting the compressive strength of rice husk ash mixed cement concrete. A two-tailed student t test with a critical value of 0.05 (α) was also employed to compare the two groups. If $t_{stat} > t_{critical}$ two-tail, we reject the null hypothesis. *Table 7* shows the experimental and model results of compressive strength for the control points. For the t test, $t_{stat} = -1.424297987$ and t critical two-tail = 2.262157163, therefore $t_{critical} > t_{stat}$. As a result, we reject the null hypothesis. *Table 8* shows the results.

If $F > F_{crit}$, the null hypothesis of the analysis of variance is rejected. *Table 9* shows the analytical results: $F = 0.046158$ and $F_{crit} = 4.413873$, indicating that $F_{crit} > F$. As a result, we do not reject the null hypothesis. This, however, implies that there was no substantial difference between the experiment and model results. As a result, the model is now suitable for predicting the compressive strength of rice husk ash mixed cement concrete.

Source of Variation	SS	df	MS	F	P-value	F_{crit}
Between Groups	2.171405	1	2.171405	0.046158	0.832304	4.413873
Within Groups	846.7743	18	47.04302			
Total	848.9457	19				

Table 10 Statistical analysis of the results
10. táblázat Az eredmények statisztikai analízise

3.4 Discussion of results

In general, the simplex technique of Scheffe was used in this research, and the outcomes of 28 days compressive strength were achieved. *Tables 5* and *6* show the compressive strength findings derived from both the laboratory response and the model response. Based on the developed model, a peak compressive strength of 40.75 N/mm^2 was obtained with a corresponding mix ratio of 0.475: 1.00: 2.75: 3.50 for the fractions of water, binder (80% cement, 20% RHA), fine aggregate, and coarse aggregate. It is worth noting that the addition of approximately 2.598 percent by weight of rice husk ash to the concrete mix with a water cement ratio of 0.475 resulted in maximum compressive strength value. The lowest compressive strength response was 7.41 N/mm^2 as a result of adding approximately 2.36 percent by weight of rice husk ash to the concrete mix with water cement ratios of 0.47: 1.00: 2.50: 4.5 for binder, fine aggregate, and coarse aggregate, respectively. The maximum 28 days compressive strength value was higher than the minimum requirements of 20 and 25 N/mm^2 cube strength of concrete for structural application NCP 1 [24] and reinforced concrete according to BS 8110: Part 1 [25]. This indicates, however, that the compressive strength model may be used to predict concrete grades C8/10 to C32/40 according to BS EN 206 [26]. Furthermore, this finding shows that RHA, although being an excellent SCM, may still be used as a building material in concrete structures in order to promote environmental protection, eliminate waste management issues, and promote sustainable development.

4. Conclusions and recommendations

Scheffe's second degree polynomial was used in this study to develop a model for optimising the compressive strength of rice husk ash blended cement concrete. The results showed that the response predicted by the formulated model corresponds well with the practically observed results. The maximum compressive strength of 40.75 N/mm^2 was obtained with a mix ratio of 0.475: 1.00: 2.75: 3.50 for the fractions of water, binder (80% cement and 20% RHA), fine aggregate, and coarse aggregate, respectively. In contrast, the minimum compressive strength response was 7.41 N/mm^2 with a matching mix ratio of 0.47: 1.00: 2.50: 4.5. Based on the test of adequacy, F-statistical tests, student t test, and analysis of variance (ANOVA) test at 95 percent confidence level were used to check the adequacy of the models, and the results show that there is a strong relationship between the control laboratory values and computed model results, with a p-value of 0.832 obtained from the ANOVA statistical results. ANOVA, on the other hand, proved to be the most appropriate approach for this objective. Furthermore, using the model equations, equivalent optimisation for any desired response within the simplex may be performed.

Notations

k	=	degree of dimensional space
q	=	number of components
n	=	order of polynomial regression
m	=	order of the Scheffe's polynomial
X_i	=	proportion of ith components of mixtures
X_1	=	proportion of water cement ratio
X_2	=	proportion of binder (cement and RHA)
X_3	=	proportion of fine aggregate
X_4	=	proportion of coarse aggregate
Z	=	actual components
X	=	pseudo components

$Y_1, Y_2, Y_3, Y_4, Y_{12}, Y_{13}, Y_{14}, Y_{23}, Y_{24}, Y_{34}$ = responses from treatment mixture proportions

$C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}$ = responses from control mixture proportions

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{23}, \alpha_{24}, \alpha_{34}$ = model coefficients

Y = optimised compressive strength of RHA concrete

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