Band structures of metacomposite based phononic crystals in quasi-Sierpinski fractals

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Abstract

In this paper, we investigated the bandgaps of two-dimensional phononic crystals with quasi-Sierpinski carpet unit cells in a metacomposite based solid–solid phononic crystal. Finite element method was used to analyze the properties of two-dimensional phononic bandgaps (2D PBGs) in a quasi-fractal structure. Two new types of quasi-Sierpinski fractal unit cells whose constituents are homogeneous and isotropic were proposed to obtain larger full bandgaps. The results show that the PBGs of the proposed quasi-Sierpinski fractals are suitable to tune the PBG's without changing the size of the phonic crystal. The new quasi-Sierpinski fractals also retain the self-similarity as in the third-order Sierpinski fractal unit cell. The investigated quasi-fractals can be easily modified to increase the filling fraction of the constituents, which can be effectively used to enlarge existing PBG by preserving degree of self-similarity structure.

Keywords: phononic crystals, metacomposite, quasi-Sierpinski carpet, bandgaps

Kulcsszavak: fononikus kristályok, metakompozit, kvázi-Sierpinski szőnyeg, sáv hézagok

1. Introduction

When an elastic wave propagates through periodic solid structures, a unique feature of periodic structures may be exhibited as phononic bandgaps (PBGs). Elastic waves with frequencies in the band gap ranges cannot propagate in the periodic structures. The existence and location of bandgaps are dependent on the material and geometrical parameters of the representative unit cell. The location and spatial distribution of bandgaps are dependent on the material, structural optimization, topology optimization, and geometrical parameters of the representative unit cell. Recently, some interesting behaviors by fractal configurations (Sierpinski carpet and Sierpinski gasket) have been discovered by researchers in recent years [1-4]. Generally, fractal systems, over microscopic to macroscopic length scales, are ubiquitous in nature. Inspired by the natural fractal, hierarchical structures/composites with self-similar fractals have been de-signed for achieving remarkable mechanical properties (high stiffness, strength, toughness). Besides the fractal- inspired hierarchical structures, periodic structures (well-known as phononic crystals) aiming at modulating wave propagation have received growing interests in the latest decade. For a phononic crystal, the frequency range within which the incident wave with any propagation direction cannot pass through is deemed to be a bandgap. It provides guidance for the design of the acoustic filter and the sound/vibration insulator by investigating the band structure of a phononic crystal [1].

Due to the unique features of fractal structures [3], most of the studies have been focused on developing strategies to obtain low, high, and wider frequency bandgaps by changing the fractal order and fractal size. However, it is a difficult task to change the distinctive characteristic of the wave propagation with the traditional Sierpinski fractals. Therefore, ongoing studies have been shifted to quasi-fractal phononic crystals to broaden the potential applications in the field of noise control and vibration reduction.

Yan et al. proposed layered phononic crystals with different fractal super-lattices and studied elastic wave localization in them [4]. Gao and Wu et al. investigated the bandgaps of two-dimensional (2D) PCs which are constituted by self-similarity shape cells [5, 6]. Norris used the finite-difference time-domain (FDTD) analysis to determine the impact of periodic fractal-shaped inclusions on the frequency response of 2D PCs. [7]. Kuo and Piazza introduced the T-square fractal geometry design for a microscale phononic BG structure in aluminum nitride and obtained ultra-high frequency bandgaps by the design of fractal phononic crystals in aluminum nitride [8, 9]. Liu et al. presented T-square fractal holes in a solid/air structure and found the origin of the lower frequency bands [10].

The PCs with periodically distributed void pores belong to a kind of light weight structure. Wang has studied the bandgap of the 2D PC with a kind of nonconvex holes [11]. Liu has studied the influence of T-square fractal shape holes on the band structure of 2D PCs [12]. This research shows that the structure with increasing fractal level of the holes results will affect the BGs.

Gao et al. obtained low-frequency BGs by studying the effects of the geometrical parameters and degree of the self-similarity structure [13].

In order to study the impact of a quasi-fractal design on the band structures of the two-dimensional phononic crystal, we propose two new types of quasi-fractal unit cells were to obtain large full bandgaps by designing two different quasi-Sierpinski carpet unit cells.

2. Unit cell with fractal pattern

We consider a type of two-dimensional metacomposite phononic crystal with its unit cell consisting of a fractal Sierpinski gasket structure. The structure is distributed in a square array with the lattice constant *a*. Therefore, we start with a square plate, as illustrated in *Fig. 1(a)*. Geometrically, the Sierpinski carpet structure that exhibits a repeating pattern at every level is a fractal topological structure. If the repeated pattern is the same as the previous one, self-similarity appears in the structure. However, a self-similar pattern in the structure at each level has a different scaling factor for the fractals with various degrees of self-similarity. In the Sierpinski gasket structure, the unit square with lattice constant a is equally divided into 9 sub squares that make up a 3×3 array of sub cells, as shown in *Fig. 1(b)*. From the array, a small central square of side length b is carved out from an elastic matrix and filled with the other material of choice. At this step, the second-order fractal, n=2, whose side length $a/3$ is obtained. Repetition of this process for the remaining sub squares gives Sierpinski carpet structure with n=3, as shown in *Fig. 1(c)*. It is seen that the side length of the unit square is reduced by 1/3 for the second-order. Applying the same rule at each iteration, subcells of lateral size $b_n = a \times (1/3)^{n-1}$ can be obtained. Briefly, the Sierpinski fractal unit cells with first-order (n=1), second-order (n=2), third-order (n=3), and fourth-order (n=4) are given in *Fig. 1(a)-(d)*, respectively. It is clear that a square structure with its side length is considered as the first-order fractal.

The fourth-order Sierpinski Carpet can be used to obtain a quasi-Sierpinski carpet. Therefore, some modifications to the fourth order Sierpinski structure can be made in order to get a new quasi-Sierpinski carpet. However, in order to attain a wider frequency band gap, modifications are employed in two different ways to utilize two new types of quasi-Sierpinski carpets. The first quasi-Sierpinski fractal structure was obtained by only connecting the squares with a side lenght of $a_o/27$ around the squares with a side lenght of $a_o/9$. *Fig. 2 (a)* and *(b)* show the example of a quasi-fractal structure and its pattern formation.

On the other hand, the second quasi-Sierpinski fractal structure was also obtained by only connecting the squares with a side lenght of $a_o/27$ along the x and y direction in a way that no sides of any other squares with a side lenght of $a_o/3$ and $a_o/9$ were cut through. *Fig. 3(a)* and *(b)* show the example of a quasi-fractal structure and its pattern formation.

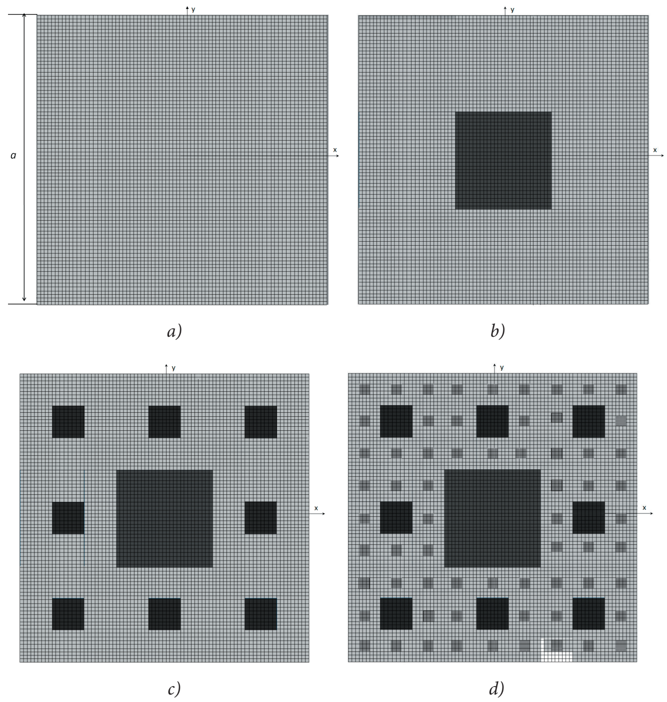


Fig. 1 Traditional Sierpinski-carpet unit cells with first-order (a), second-order (b), third-order (c) and fourth-order (d, respectively)
1. ábra Elsőrendű (a), másodrendű (b), harmadrendű (c) és negyedrendű (d) hagyományos Sierpinski-szőnyeg egységcellák

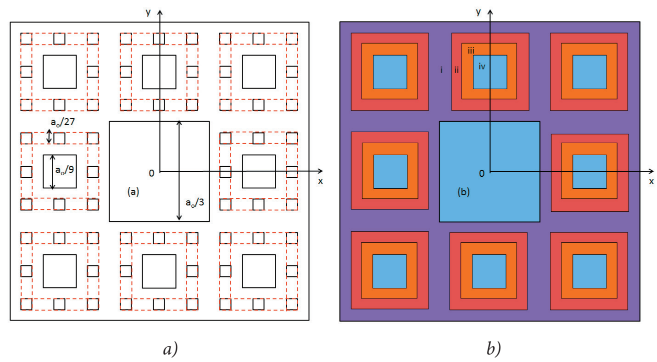


Fig. 2 Quasi-Sierpinski carpet unit cell (a) and its pattern formation (b)
2. ábra Kvázi-Sierpinski szőnyeg egység cella (a) és mintázatának kialakítása (b)

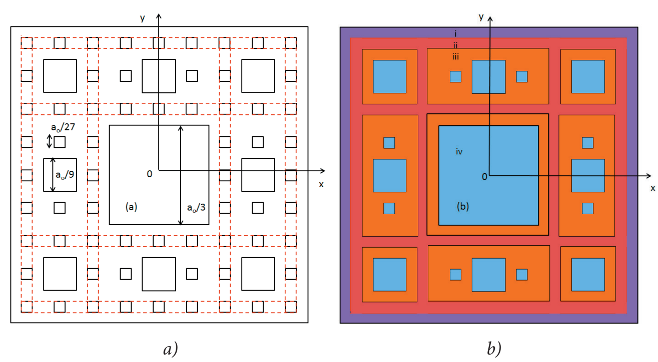


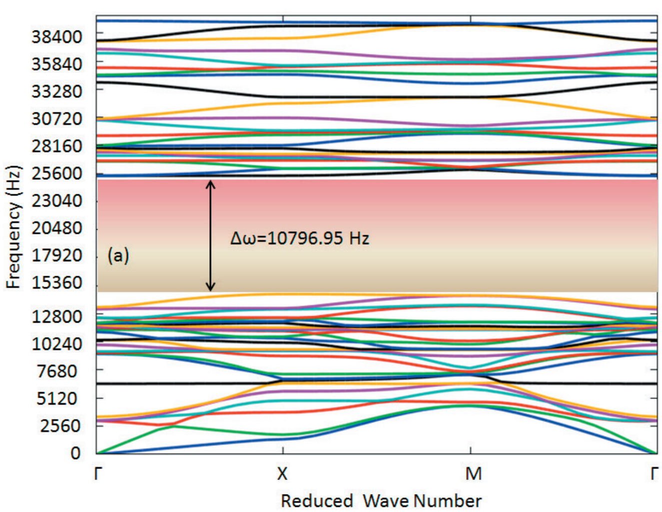
Fig. 3 Quasi-Sierpinski carpet unit cell (a) and its pattern formation (b)
3. ábra Kvázi-Sierpinski szőnyeg egység cella (a) és mintázatának kialakítása (b)

The unit cell of both quasi-Sierpinski fractal structures consists of a center square as in the 2nd order Sierpinski carpet with six other squares repeated as in the 3rd order repeating at its corners. Due to the specific placement of the square rods, these two quasi fractal structures still retain their symmetry. These particular arrangements give rise to four different regions within the structure. These four different regions are shown in *Fig. 2 (b)* and *3 (b)*. This can allow us to consider the structure as a four-component material structure. As a result of the obtained geometry, the elastic matrix was filled with four different materials. Suggesting such structures with the self-similar arrangement also show that the fractals behave as phononic quasi-crystals.

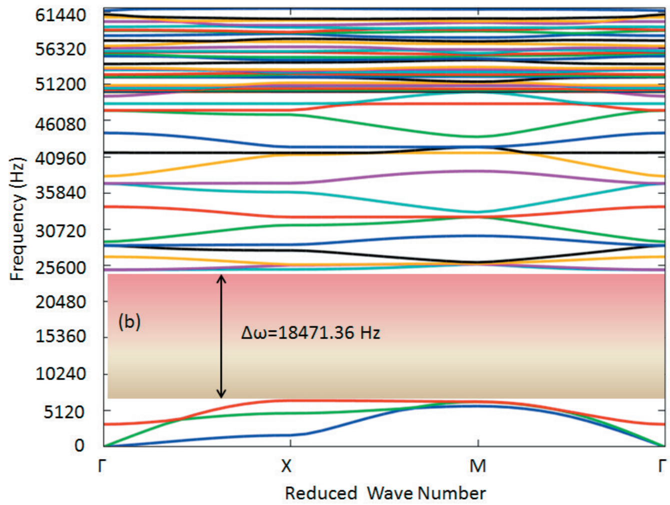
The unit cell, which comprises region I, region II, region III, and region IV, are filled with matrix material silicon rubber composite, $LiNbO_3$ inclusions, PZT($Pb[Zr_xTi_{1-x}]O_3$ - Lead Zirconate Titanate) inclusions and porous Si_3N_4 composite inclusions, are arranged in a square lattice with the lattice constant a =1.5 mm. Density ρ, Elastic modulus E, and Poisson's ratio $\boldsymbol{\nu}$ of rubber, PZT and porous Si_3N_4 composite are ρ_1=1300 kg/m^3 E_1=1.175x10^5 [Pa] ν_1=0.4688, ρ_2=11600 kg/m^3 E_2=40.819x10^9 [Pa] $\boldsymbol{\nu}_2$=0.3698 and ρ_3=17800 kg/m^3 E_3= 3.6x10^{11}[Pa] ν_3=0.28, respectively [14]. Parameters of piezoelectric material properties of PZT and $LiNbO_3$ were used as default parameters defined by the COMSOL® software.

3. Results and discussion

In order to understand the variation of the two quasi-Sierpinski carpets PBG, the Finite Element Method (FEM) based on the COMSOL® simulation software with the solid mechanics module was used to find the band structures. The bandgap structures of in-plain modes for *Fig. 2* and *3* are given in *Fig. 4(a)* and *(b)*. There are absolute bandgaps that appeared as shown in both figures along with Γ-X, Γ-M and X-M directions. *Fig. 4(a)* shows a wide full bandgap ranging from 14622.91 Hz to 25919.86 Hz. However, a larger full band gap appeared in the frequency range from 6561.301 Hz to 25032.66 Hz, *Fig. 4(b)*. Since the conventional Sierpinski carpets are difficult to significantly adjust the band gap behavior because the filling fraction is small. The difference between the full bandgaps is probably due to the increased filling ratio as well as the difference between the structures. *Fig. 4(a)* and *(b)* show that proposed the quasi-Sierpinski carpet phononic crystals have a wider and expanded vibration isolation capacity.



a)



b)

Fig. 4 Band structures of the PC structures, (a) and (b), as described in Fig. (2) and (3), respectively

4. ábra A (2) és (3) ábrák szerint PC-struktúrák (a) és (b) sáv hézagai

The reflection and refraction of a certain wave can be analyzed in the space of wave numbers by considering the equifrequency surface (EFS) of the wave. This surface is described by the dispersion relation of the anisotropic medium at the fixed frequency ω. The EFSs of the incident wave along the Γ–X–M–Γ direction are shown in *Figs. 5(c-d)*. *Figures 5(s-d)* show that local curvature of the equi-frequency contours deviates from circular symmetry. The circular contours are distorted and become a rectangular-like contour by leading to a simple symmetry for values of k_x equal to k_y. An equi-frequency plot of the 2-nd band, which is a little bit complicated than the 1-st band exhibits a square-like contour. The case of a non-circular equi-frequency contour, however, indicates anisotropic behavior.

On the other hand, a square-like contour also reflects some symmetry in a particular direction. This could especially be advantageous and even be more effective if the self-collimation property of the phononic crystal is considered. It must be noted that the group velocity v_g is normal to the equi-frequency contour but not collinear to the wave vector k. At each frequency, the energy flow direction is given by the normal to the equi-frequency contour, and is in the direction of the maximum rate of change of frequencies. The calculated contours also allow us to analyze whether the phononic crystal can have negative refraction or not, depending on the sign of the dispersion slope. From the equi-frequency contour plots above, we can see that the radius of the circle increases with frequency. Hence, we can conclude that the dispersion slope is positive. In order to understand the dynamics of wave propagation, the concept of group velocity may be useful from the viewpoint of energy transportation. Therefore, the group velocity is the speed, with which a wave packet moves, along the high-symmetry directions Γ–X–M of the Brillouin zone. It is worth to mention that the group velocity of waves are zero at the some high symmetric points (X or M) meaning that there is no energy transfer at these points (*Figs. 5a* and *5b*).

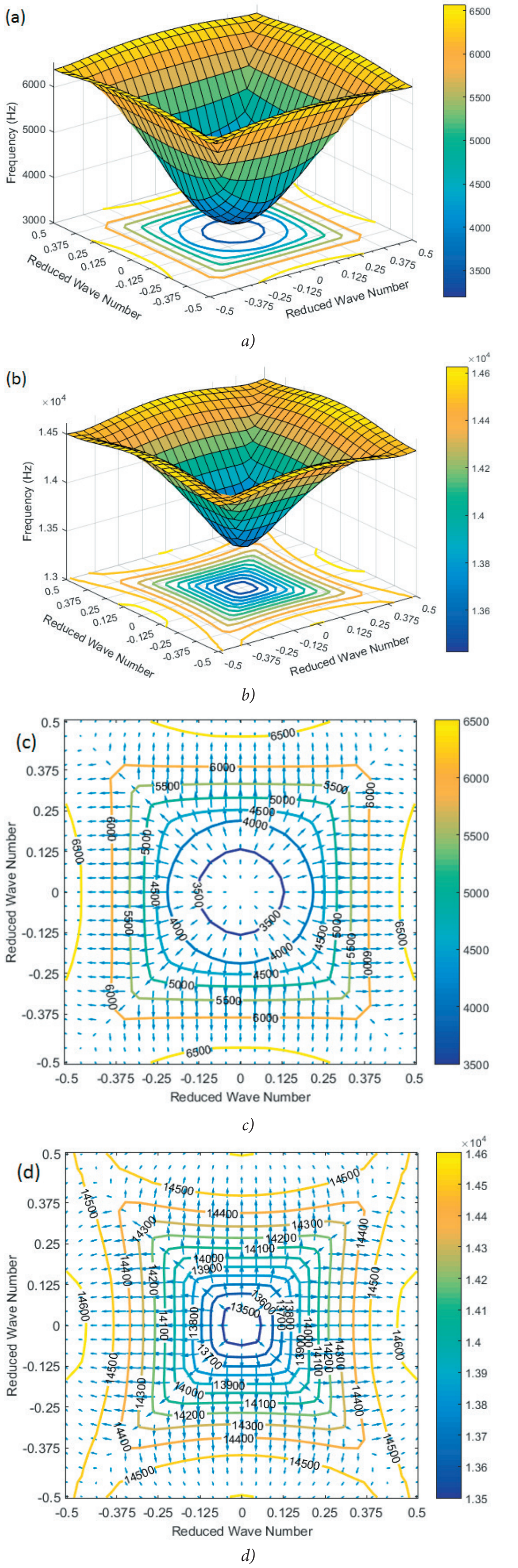


Fig. 5 The dispersion relation ω(k), for the 27th and 3rd modes for all k vectors in the first Brillouin zone, (a-b). Corresponding equi-frequency contours for the 27th and 3rd modes, (c-d)

5. ábra Az ω(k) diszperziós összefüggés a 27. és 3. módusra, az első Brillouin zónában lévő összes k vektor szerint, (a-b). Megfelelő ekvi-frekvenciás kontúrok a 27. és 3. módusokhoz, (c-d)

4. Conclusions

The FEM method was applied to investigate the dispersion curves of phononic crystals with quasi-Sierpinski carpet unit cells. Two new quasi-Sierpinski carpets were presented and the results show absolute bandgaps. The influence of the different configurations of the 4th order Sierpinski carpet on the bandgaps was demonstrated. The proposed quasi-Sierpinski carpets can be used to attain larger bandgaps than those observed with traditional Sierpinski carpets. Therefore, the appropriate modifications by retaining self-similarity to some degree and suitable fractal order of Sierpinski carpet structure can be utilized to extend vibration-free zones without changing the entire size of the phononic crystal. Compared to the traditional Sierpinski fractals in a regular configuration, it is also easier to increase the filling fraction to observe the band gap features.

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