The analysis of composite piezo-magnetic beams into dynamic nonlocal nonlinear case

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Abstract
This paper investigates the nonlinear dynamical behaviour of piezo-magnetic beams using the formulation by a refined higher-order beam that incorporates nonlocal effects and the piezoelectric phase. The increase in the piezoelectric phase can enhance the vibrational behaviour of bright beams under magnetic fields and harmonic excitation. We have derived the nonlinear governing equations for a nonlocal intelligent beam based on the refined beam model, and a numerical method has been introduced to calculate the nonlinear vibrational curves. Our study shows that variations in the volume fraction of the piezoelectric material significantly impact the vibrational behavior of the intelligent nanobeam when exposed to electrical and magnetic fields. Additionally, the nonlinear free and forced vibrational behaviour of the intelligent nanobeam is influenced by the magnitudes of induced electrical voltages, magnetic potential, the stiffness of the elastic substrate, and shear deformation.

Keywords: composite, dynamic behaviour, forced vibration, vibration; piezoelectric, nonlocal
Kulcsszavak: kompozit, dinamikus viselkedés, kényszerrezgés, szabad vibráció; piezoelektromos erősítés, nem lokális elmélet

1. Introduction

Recent advancements in engineering materials have highlighted the benefits of smart or intelligent materials [1]. The incorporation of these innovative materials into various multifunctional structures has led to significant changes across different engineering fields. Among these materials, magneto-electro-elastic (MEE) materials stand out because they exhibit a unique triple energy change between elastic, electrical, and magnetic arenas [2-4]. As a result, they are promising candidates for sophisticated applications such as vibration control, sensors, energy harvesting, and actuators. Recently, researchers have focused on synthesising MEE structures with composite materials to enhance their structural functions. For example, the properties of multi-phase MEE materials might be worked by varying the composition and proportion of each phase [5,6]. Recognising the significance of intelligent structures made from MEE materials with different compositions in industrial contexts, many scholars have dedicated their research to evaluating mechanical responses in various operational environments [7, 8].

At the nanoscale, size effects have a significant impact on both physical and mechanical properties. This phenomenon has prompted researchers to investigate the mechanical response of nanostructures. The fundamental limitation of classical continuum mechanics is its inadequacy in modeling small-scale structures, which has led to the development of higher-order continuum theories that easily incorporate size dependency [9-14]. Eringen's nonlocal elasticity theory [15] has proven useful in addressing size effects. Due to the challenges associated with experimentally testing nanoscale structures, many articles have been published to optimize the use of this theory in assessing size-dependent structural responses [16-19]. These studies indicate that, with a higher nonlocal parameter value, nonlocal elastic models are effective in yielding a stiffness-softening effect.

Building on Eringen's nonlocal elasticity theory, some researchers have analysed MEE or piezo-magnetic nanostructures. For example, Ke and Wang [20] studied the linear vibrational properties of intelligent nanoscale beams using nonlocal theory. Additionally, Jandaghian and Rahmani [21] investigated the linear vibrational characteristics of intelligent nanoscale beams on elastic foundations. Ebrahimi and Barati [22] also examined the vibrational properties of a functionally graded intelligent nanoscale beam using nonlocal theory. In light of these findings, this article aims to develop a multi-phase MEE nanobeam resting on a nonlinear elastic substrate for dynamic analysis within the framework of nonlocal elasticity theory. An approximate solution is then provided based on Galerkin's technique. A parametric study is conducted to study the effect of nonlocality, various piezoelectric volumes, electromagnetic fields, and elastic substrate coefficients on the structural performance of these nanoscale systems. The results of this research have the potential to significantly impact the design and optimisation of innovative structures under dynamic loads, offering promising prospects for future research and development in this field.

At the nano range, the significant effect of size is noticed on both mechanical and physical properties. This case has interested a few authors in distracting their focus from assessing the mechanical response of the nanostructures. The main limitation of classical continuum mechanics is its disorganisation in modelling small-size structures, which paved the way for the founding of higher-order continuum theories which join the size dependence of structure with ease [10-14]. Eringen's nonlocal elasticity theory [15] proved to be handy in employing the side effects. Because experimenting with nano-size structures is still hard, numerous articles have been available to best utilise this theory in evaluating size-dependent structural responses [16-19]. The primary result of these authors indicates that the higher value of the nonlocal parameter and nonlocal elastic models are efficient and sufficient only to yield a stiffness-softening effect. Joining Eringen's nonlocal elasticity theory, few authors tried to analyse the MEE or piezo-magnetic nanostructures. With the usage of nonlocal theory, a study on linear vibrational properties of intelligent nano-size beams has been represented by Ke and Wang [20]. Moreover, Jandaghian and Rahmani [21] represented linear vibrational investigation of intelligent nano-size beams based on elastic foundations. In another research, Ebrahimi and Barati examined the vibrational properties of a functionally graded intelligent nano-scale beam using nonlocal theory [17].

This article aims to develop a multi-phase magnetoelectric (MEE) nanobeam that rests on a nonlinear elastic substrate, focusing on dynamical analysis within the framework of nonlocal elasticity theory. We propose that the MEE composite consists of two phases, each containing piezoelectric and magnetic components. To investigate the nanoscale effects, we employ Eringen's elasticity theory.

The authors in this study derive the equilibrium equations of the nanobeam with properties of MEE by using Hamilton's method and von Kármán geometric nonlinearity. An approximate solution is then obtained using Galerkin's technique. Furthermore, a comprehensive parametric study is conducted to examine how nonlocality, various piezoelectric volume percentages, electromagnetic field effects, and elastic substrate coefficients influence the structural performance of these nanoscale systems. The findings of this paper have the potential to significantly impact the design and optimisation of innovative structures under dynamic loads, paving the way for future research and development in this promising field.

2. Two-phase composite

Fig. 1 illustrates a nanoscale beam made of a magneto-electro-elastic composite consisting of two phases. The material properties of this multi-phase magneto-electro-elastic (MEE) composite depend on the ratio and volume of the piezoelectric phase (Vf). This item focuses on a nanobeam constructed from a composite of $BaTiO_3$ and $CoFe_2O_4$. In this context, $BaTiO_3$ serves as the piezoelectric component, while $CoFe_2O_4$ acts as the piezomagnetic component.

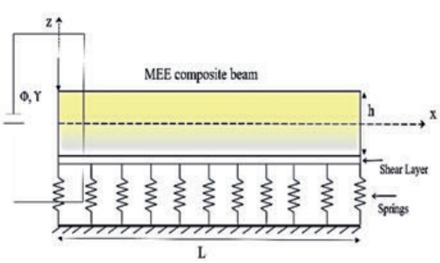

Fig. 1 A composite nanobeam rested on an elastic substrate
1. ábra Rugalmas anyagon nyugvó kompozit nanogerenda

3. Formulation due to refined beam theory

Different beam and plate theories are available in the literature [23-37]. This section will present the procedure for deriving the governing equations for a piezo-magnetic nanobeam within the framework of both nonlocal and classical beam theories. To achieve this, the displacement field of a nanoscale beam, based on axial (u) and transverse (w) displacements at the mid-axis, can be expressed as follows (Fourn et al. 2018):

$$u_1(x,z,t) = u(x,t) - z\frac{\partial w_b}{\partial x} - (z)\frac{\partial w_s}{\partial x} \tag{1}$$

$$u_2(x,z,t) = w(x,t) = w_b(x) + w_s(x) \tag{2}$$

Based on *Equations 1* and *2*, the authors applied mathematical treatments to derive the following equations:

Next, integrating *Equation 3* yields:

$$\frac{\partial u}{\partial x} = -\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{A_{31}^e}{A_{11}}\phi - \frac{A_{31}^m}{A_{11}}\gamma + \frac{B_{11}}{A_{11}}\frac{\partial^2 w_b}{\partial x^2} + \frac{B_{11}^s}{A_{11}}\frac{\partial^2 w_s}{\partial x^2} + \frac{N_x^E}{A_{11}} + \frac{N_x^H}{A_{11}} + \frac{C_1}{A_{11}} \tag{3}$$

So, Satisfying edge settings; u(0)=0, u(L)=0, one can derive:

$$u = -\frac{1}{2}\int_0^x \left(\frac{\partial w}{\partial x}\right)^2 dx - \frac{A_{31}^e}{A_{11}}\int_0^x \phi dx - \frac{A_{31}^m}{A_{11}}\int_0^x \gamma dx + \frac{B_{11}}{A_{11}}\frac{\partial w_b}{\partial x} + \frac{B_{11}^s}{A_{11}}\frac{\partial w_s}{\partial x} + \frac{N_x^E}{A_{11}}\int_0^x dx + \frac{N_x^H}{A_{11}}\int_0^x dx + \frac{C_1}{A_{11}}x + C_2 \tag{4}$$

$$C_2 = -\left(\frac{B_{11}}{A_{11}}\frac{\partial w_b}{\partial x} + \frac{B_{11}^s}{A_{11}}\frac{\partial w_s}{\partial x}\right)_{x=0}$$

$$C_1 = \frac{A_{11}}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx + \frac{A_{31}^e}{L}\int_0^L \phi dx + \frac{A_{31}^m}{L}\int_0^L \gamma dx - \left(\frac{B_{11}}{L}\frac{\partial w_b}{\partial x} + \frac{B_{11}^s}{L}\frac{\partial w_s}{\partial x}\right)_{x=L} - (N_x^E + N_x^H) \tag{5}$$

As the next step, finded constant must be situated in *Equation 50.*

4. Solution method

In this study, the governing equations of motion for free/forced vibrations of simply-supported MEE nano-size beams have been solved by employing Galerkin's approach. The displacement functions are provided as a creation of non-unknown coefficients and identified trigonometric functions to promise the conditions of boundary at x=0 and x=L as [26]:

$$w_b = \sum_{p=1}^{\infty} W_{bp}(t) X_p(x) \tag{6}$$

$$w_s = \sum_{p=1}^{\infty} W_{sp}(t) X_p(x) \tag{7}$$

$$\phi = \sum_{p=1}^{\infty} \Phi_p(t) X_p(x) \tag{8}$$

$$\gamma = \sum_{p=1}^{\infty} \Upsilon_p(t) X_p(x) \tag{9}$$

where (W_{bp}, W_{sp}, Φ_p, γ_p) display the field largest values and the function $X_p = \sin(p\pi x/L)$ displays the shape function of the simply supported beam ($w = \frac{\partial^2 w}{\partial x^2} = \gamma = \phi = 0$). Placing *Equations 6–9* in governing equations yields below equations:

$$K_{1,1}^S W_{bp} + K_{2,1}^S W_{sp} + G_1 W_p^3 + Q_1 W_p^2 + M_1 \ddot{W}_p + K_1^E \Phi_p + K_1^H \Upsilon_p = F\cos(\omega t)$$
$$K_{1,2}^S W_{bp} + K_{2,2}^S W_{sp} + G_2 W_p^3 + Q_2 W_p^2 + M_2 \ddot{W}_p + K_2^E \Phi_p + K_2^H \Upsilon_p = F\cos(\omega t)$$
$$K_{1,3}^S W_{bp} + K_{2,3}^S W_{sp} + G_3 W_p^2 + K_3^E \Phi_p + K_3^H \Upsilon_p = 0$$
$$K_{1,4}^S W_{bp} + K_{2,4}^S W_{sp} + G_4 W_p^2 + K_4^E \Phi_p + K_4^H \Upsilon_p = 0 \qquad (10)$$

By using KS as components of the stiffness matrix, Gi as stiffness of nonlinear . The overhead equations are concurrently resolved in order to get nonlinear vibration frequencies. Then the approximate solution has the below definition:

$$W_p(t) = \tilde{W}\ \cos(\omega t) \qquad (11)$$

Also, dimensionless quantities are selected as:

$$K_L = \mathrm{k}_L \frac{\mathrm{L}^4}{D_{11}},\ K_p = \mathrm{k}_p \frac{\mathrm{L}^2}{D_{11}},\ K_{NL} = \mathrm{k}_{NL} \frac{\mathrm{L}^4}{A_{11}}$$
$$\tilde{\omega} = \omega L^2 \sqrt{\frac{\rho A}{\tilde{c}_{11} I}}, \mu = \frac{ea}{L}, \tilde{F} = F\frac{L^2}{A_{11} h} \qquad (12)$$

5. Numerical results and discussions

In this section, we have presented several graphical examples and discussed the results obtained to validate the free vibrational properties of multi-phase MEE nano-sized beams. The results, based on the assumption of a geometrically perfect nanobeam, have been provided. The length of the nano-scale beam is set at L = 10 nm. To ensure the reliability of our approach, we have compared our findings with the work of Li et al. [16] regarding the non-linear vibration frequencies of imperfect nanobeams based on various maximum vibration amplitudes. This comparison serves to affirm the robustness of our model, as our results align with those reported by Li et al. [16].

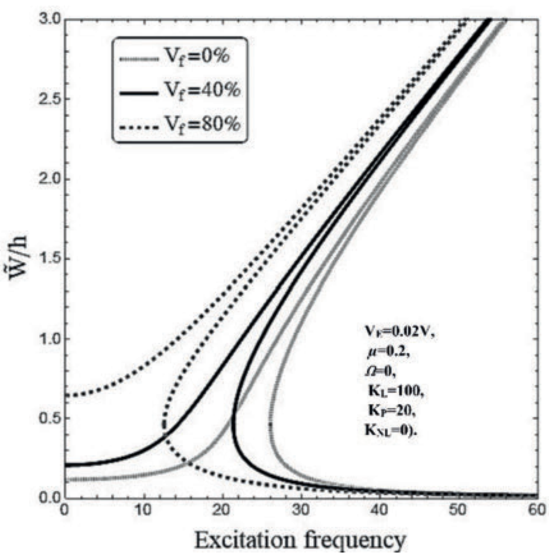

Fig. 2 Impact and voltage on vibration frequency curves of the nanobeam
2. ábra A nanogerenda rezgési frekvenciájára gyakorolt hatás és feszültség

The influence of piezoelectric volume on forced vibrational curves of the nanobeam is shown in *Fig. 2,* $\tilde{F}$=0.01. The volume of piezoelectric ingredients has been selected to be V_f=0%, 40% and 80%. From the figure, it may be understood that enhancing the volume of piezoelectric ingredients yields lower shift frequencies. This is associated with the decrement in the elastic stiffness of nano-scale beams by increasing the piezoelectric portion. Afterwards, the elastic modulus of composites decreases by increasing the piezoelectric ingredient. Also, as the magnitude of electric voltage is lower, the curves are closer to each other. Accordingly, an MEE nano-scale beam with higher percentages of piezoelectric ingredients is more susceptible to induced electrical fields.

Fig. 3 provides a comparison among non-linear frequencies based upon classical and improved (refined) shear deformation beam types of MEE nano-sized beams. The presented graph has been illustrated according to the hypothesis that the aspect ratio is L/h=10. This figure highlights that non-linear vibrational curves tend to have higher frequencies when the magnitudes of non-dimension deflection grow. Such observation is associated with stiffening influences of non-linear geometrical factors. Moreover, one may understand that improved beam theory grants more minor non-linear vibrational frequencies than classical theory because of the impact of the insertion of shear deformation. Hence, the improved theory is more reliable for a thick intelligent piezoelectric-magnetic beam.

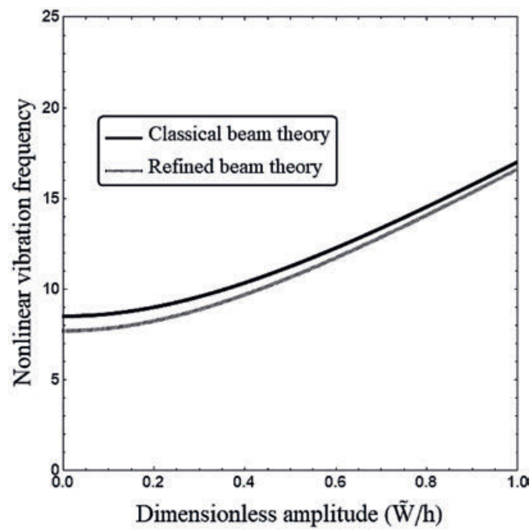

Fig. 3 a comparison among non-linear frequencies of MEE nano-sized beam
3. ábra Az MEE nano-méretű gerenda nemlineáris frekvenciáinak összehasonlítása

Fig. 4 illustrates the effect of tiny scales on the non-linear vibrational frequency of a two-phase magnetoelectric (MEE) nanoscale beam in relation to normalised vibrational amplitudes. It is evident that as the parameter of dimensionless nonlocal (μ) increases, the normalised frequency decreases. This indicates that classical elastic theory, which does not account for small-size effects, tends to provide higher approximations for normalised vibrational frequency. In contrast, nonlocal continuum mechanics offers greater precision and reliability, resulting in more accurate outcomes.

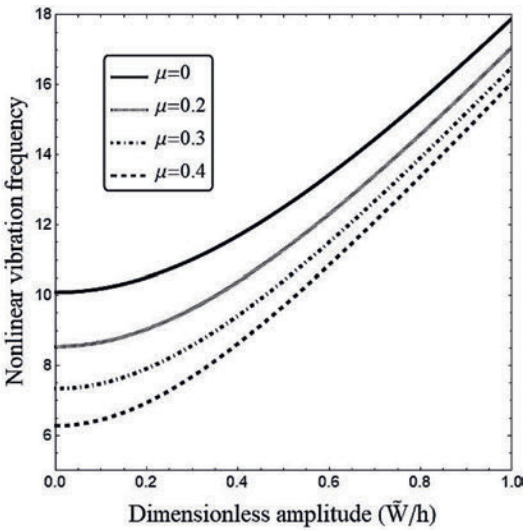

Fig. 4 The effect of small scales on the non-linear vibrational frequency
4. ábra A kis méretek hatása a nemlineáris rezgési frekvenciára

Fig. 5 displays changes in non-linear vibration frequency versus normalised amplitude under various electric voltages (VE) and magnetic field intensities (Ω). Notably, the non-linear shift frequency decreases as the applied electric field transitions from negative to positive voltages. Similarly, an increase in magnetic field intensity from negative to positive results in a rise in non-linear vibration frequency. This phenomenon can be attributed to the exceptional capacity of MEE materials to absorb and maintain magnetism. As the intensity of the magnetic field increases, this ability becomes more pronounced, facilitating the conversion of magnetic potential into mechanical force. Consequently, as the magnetic field exerts tensile forces on the nanobeam, the non-linear vibration frequency increases.

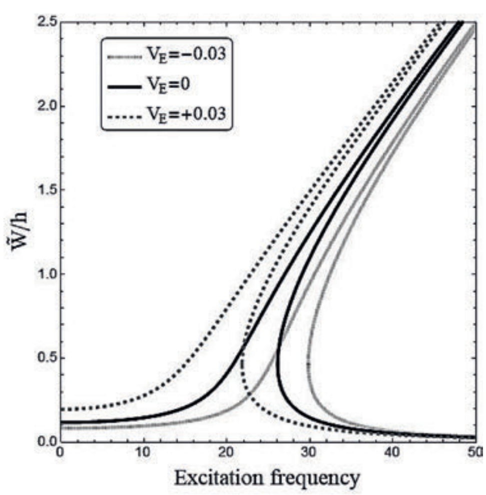

Fig. 5 The changes in non-linear vibration frequency
5. ábra A nemlineáris rezgési frekvencia változásai

6. Conclusion

This research investigates the fascinating dynamics of nonlocal, nonlinear free and forced vibrations in two-phase magneto electro elastic (MEE) nanobeams, presenting an insightful analytical perspective. These nanobeams are modeled to rest on an elastic foundation characterized by three key parameters: linear, shear, and nonlinear.

Notably, our findings reveal that as the dimensionless nonlocal parameter increases – indicating the significance of nonlocal effects – the normalized frequency, which is independent of system size, correspondingly decreases. This clearly illustrates that the classical elastic model, which neglects small-scale effects, produces inflated estimates of the non-dimensional vibrational frequency.

Moreover, the study highlights an intriguing trend: the nonlinear foundation parameter significantly influences vibration frequency curves, particularly as vibration amplitude rises. It is also crucial to note that the interaction of the magnetic field with the vibration characteristics of MEE nanobeams is contingent upon the piezoelectric volume. Yet, as the piezoelectric volume increases, the rate of frequency enhancement in response to magnetic field intensity diminishes. This underscores the complexity and interdependence of these parameters, reinforcing the importance of considering them in future research.

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