Predictive modeling of concrete split tensile strength using Scheffe's simplex lattice design with reclaimed asphalt pavement as coarse aggregates

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Abstract

This study employs Scheffe's Simplex Lattice Design to predict and optimize the split tensile strength of concrete incorporating reclaimed asphalt pavement (RAP) as a partial replacement for natural coarse aggregates. A {5,2} augmented Simplex Lattice, executed in Minitab 22, generated 21 experimental runs to evaluate the effects of cement, sand, water, natural coarse aggregates, and RAP. Pseudo-components were transformed into real ratios using a matrix-based approach, ensuring accurate mixture proportion representation. A quadratic regression model, with an R^2 of 92.89% and a significant F-value of 0.98 (p = 0.049), demonstrated strong predictive accuracy. All main components exhibited significant effects, with low variance inflation factors (VIF ≈ 1.59) indicating minimal multicollinearity. Optimal split tensile strength (3.14 N/mm²) was observed in a sand-dominated mixture (Run 18), closely matching the predicted 3.118 N/mm². Experimental results highlighted that a 75% RAP replacement maximized 28-day split tensile strength (3.05 N/mm²), suggesting RAP's viability as a sustainable aggregate. Residual analysis and a lack-of-fit p-value of 0.032 confirmed model adequacy. These findings offer a robust framework for optimizing concrete mixtures, advancing sustainable construction practices and predictive modeling in civil engineering.

Keywords: concrete, split tensile strength, scheffe's simplex lattice, reclaimed asphalt pavement, mixture design, predictive modeling

Kulcsszavak: beton, hasító-húzószilárdság, Scheffe-féle szimplex rács, újrahasznosított aszfaltburkolat (RAP), keveréktervezés, prediktív modellezés

1. Introduction

The split tensile strength of concrete is a critical parameter influencing the durability and structural integrity of concrete members subjected to tensile stresses [1]. Unlike compressive strength, which is more commonly studied, tensile strength is often more challenging to predict due to the heterogeneous and brittle nature of concrete. Accurate modeling of split tensile strength is essential for the design and optimization of concrete mixes, especially when incorporating alternative materials [2]. These materials can alter the microstructure and mechanical behavior of concrete, necessitating robust predictive tools to ensure performance consistency [3], [4].

Scheffe's simplex lattice design has been widely adopted as an effective statistical tool for modeling and optimizing concrete properties involving multiple mixture components. This approach allows researchers to systematically explore the effects of varying proportions of constituents on mechanical properties, including strength parameters [5], [6]. For instance, prior studies have successfully employed Scheffe's method to model compressive strength and modulus of rupture in concretes incorporating waste materials, illustrating the model's adaptability and predictive power [6], [7]. However, there remains a significant research gap

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in the application of this method specifically to split tensile strength, which exhibits different failure mechanisms and sensitivities compared to compressive or flexural strength.

The limited predictive models addressing split tensile strength often fail to capture the complex interaction effects between cement, aggregates, and supplementary materials, leading to suboptimal mix designs and potential structural inefficiencies [8]. Furthermore, variability in waste material properties and curing conditions poses additional challenges to model generalizability [9]. Thus, applying Scheffe's simplex lattice design to develop a predictive model for split tensile strength represents a promising avenue for advancing concrete technology, enabling more sustainable material use without compromising tensile performance [10].

This study aims to fill the existing gap by employing Scheffe's simplex lattice design to develop an accurate and reliable predictive model for concrete split tensile strength. By analyzing the combined influence of mix constituents and validating the model with experimental data, this research seeks to enhance mix proportioning strategies, contributing to the broader goal of sustainable and high-performance concrete construction.

2. Design of experiments using Scheffe's simplex lattice

2.1 Scheffe lattice

The design of experiments using Scheffe's method in Minitab to analyse the influence of various components (cement, fine aggregates, water, natural coarse aggregates, and reclaimed asphalt pavement (RAP)) on concrete properties was structured with the following steps:

Components:
1. Cement
2. Fine aggregates (Sand)
3. Water
4. Natural coarse aggregates
5. Reclaimed asphalt pavement (RAP)

Each step was outlined to systematically assess the impact of these constituents on the properties of concrete. The experimental setup provided a structured approach to comprehensively evaluate the effects and interactions among these essential elements.

2.2 Degree of polynomial

The degree of polynomial refers to the number of terms in the model. Generally, for Scheffe's Simplex Lattice Regression, a quadratic model (degree = 2) is a good starting point to capture non-linear effects and interactions between factors.

2.3 Selection of single total for the total mixture amount in Minitab

In the Design of Experiments (DOE) using Scheffe's method in Minitab 22, a single total for the mixture amount was chosen to ensure focus on relative proportions rather than absolute amounts, facilitating accurate assessment of component effects and interactions.

In Scheffe's method for DOE, the formula used to ensure focus on relative proportions rather than absolute amounts is:

$$\sum_{i=1}^{n} X_i = 1 \tag{1}$$

Where:

x_i represents the proportion of the ith component in the mixture.
n denotes the total number of components in the mixture.

The summation of all x_i values equals a constant, which was one (1) and was used throughout this research.

Mathematically, the sum of the proportions of the components in the mixture can be represented as:

$$x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n = 1 \tag{2}$$

Where:

$x_1, x_2, \ldots\ldots\ldots, x_n$ represent the proportions of the individual components in the mixture.

n denotes the total number of components in the mixture.

The sum of all x_i values equals a constant which is one (1).

2.4 Randomisation of the Design

In the Design of Experiments (DOE) using Scheffe's method in Minitab 22, the design was randomised. Randomisation was a fundamental principle in experimental design that helped ensure the validity of the results [11]. By randomising the order in which the experimental runs were performed, potential biases and the impact of uncontrolled factors could be minimized [12]. This was particularly crucial in mixture experiments, where the physical properties of components might vary across batches or over time. Randomising the design ensured that these variations were evenly distributed across the experimental runs, facilitating the identification of true component effects and interactions. Consequently, the randomisation of the design in Minitab 22 was a critical step in guaranteeing the reliability and accuracy of the results. The experimental design matrix for the {5,2} augmented Simplex Lattice outlines and the arrangement of pseudo components used in the study is given in *Table 1*.

Minitab 22 Experimental Parameters				Pseudo Components				
StdOrder	RunOrder	PtType	Blocks	X_1	X_2	X_3	X_4	X_5
16	1	0	1	0.2	0.2	0.2	0.2	0.2
20	2	-1	1	0.1	0.1	0.1	0.6	0.1
15	3	1	1	0	0	0	0	1
17	4	-1	1	0.6	0.1	0.1	0.1	0.1
21	5	-1	1	0.1	0.1	0.1	0.1	0.6
9	6	2	1	0	0.5	0	0	0.5
5	7	2	1	0.5	0	0	0	0.5
14	8	2	1	0	0	0	0.5	0.5
8	9	2	1	0	0.5	0	0.5	0
2	10	2	1	0.5	0.5	0	0	0
19	11	-1	1	0.1	0.1	0.6	0.1	0.1
1	12	1	1	1	0	0	0	0
18	13	-1	1	0.1	0.6	0.1	0.1	0.1
12	14	2	1	0	0	0.5	0	0.5
3	15	2	1	0.5	0	0.5	0	0
6	16	1	1	0	1	0	0	0
11	17	2	1	0	0	0.5	0.5	0
10	18	1	1	0	0	1	0	0
13	19	1	1	0	0	0	1	0
7	20	2	1	0	0.5	0.5	0	0
4	21	2	1	0.5	0	0	0.5	0

Table 1   *Experimental Design Matrix for {5,2} augmented Simplex Lattice (Pseudo Components*
1. táblázat   *Kísérleti terv mátrix a {5,2} kiterjesztett szimplex-rács tervhez (pszeudokomponensek)*
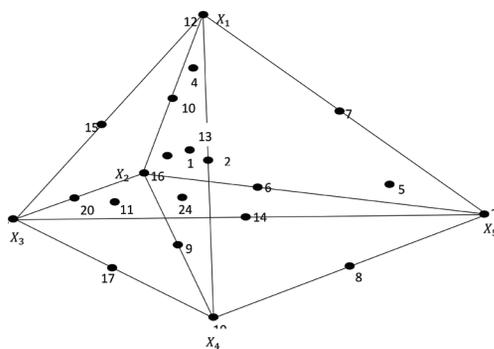
Figure 1 Scheffe's simplex lattice structure with 21 experimental runs
1. ábra   Scheffe-féle szimplex rácsszerkezet 21 kísérleti futtatással

In this simplex lattice design, 21 experimental runs were systematically arranged to evaluate the impact of various components on concrete properties, as shown in *Fig. 1*. The dataset was generated using the Scheffe Lattice design feature in Minitab 22. Each row represents a specific experimental condition, while the columns correspond to different parameters: StdOrder, indicating the standard numbering of the experiments; RunOrder, the sequence in which experiments were conducted; PtType, a categorical identifier classifying the experiments; Blocks, which groups the experiments into categories; and X_1 through X_5, representing the different pseudo-components or factors tested. The values in columns X_1 to X_5 denote the proportions or levels of each pseudo-component for each experimental run. The PtType column categorizes points into different types (-1, 0, 1, 2), reflecting various conditions or treatments applied. Each point thus corresponds to a unique combination of the pseudo-components X_1 to X_5, forming a distinct experimental setup for analysis. This structured arrangement follows the Scheffe Lattice methodology, designed to efficiently explore a broad range of component mixtures while minimizing the total number of experiments required.

2.5 Experimental parameters and pseudo components of pure blend at vertices

The experimental parameters for the pseudo components of the pure blend at the vertices are given in *Table 2*. Each row represents a unique experimental run, and the columns provide information on the run order, point type, and concentrations of the five pseudo components (X_1 to X_5) used in the experiment.

A compilation of several trial mixes derived from practical experience is given in *Table 2*. Five distinct component mix ratios were selected to represent the five vertices of the experimental design. The values corresponding to Water (X_1), Cement (X_2), Sand (X_3), RAP (X_4), and CA (Coarse Aggregates) (X_5), are the result of these selected mix ratios. Each row corresponds to a specific vertex in the experimental design, and the values reflect the relative proportions of the components in each mix, as determined through experimentation.

RunOrder	Vertex	Water	Cement	Sand	RAP	CA
12	X_1	0.6	1	1	1.5	3
16	X_2	0.55	1	2.0	1.5	3
18	X_3	0.65	1	2.5	2.0	3.5
19	X_4	0.4	1	1	2	2.5
3	X_5	0.45	1	1.5	1	2

Table 2 Component real mix ratios of the vertices of the simplex lattice
2. táblázat A szimplex rács vektorainak valós komponens keverés arányai

2.6 Component transformation analysis (converting pseudo to real ratios in experimental setup)

The process of translating pseudo components into real ratios within an experimental context is a fundamental aspect of understanding the intricate relationships between varying components in an experimental setup [13]. This transformation, often accomplished through matrix operations, offers a bridge between the observed pseudo ratios and the actual proportions of different constituents involved in an experiment [14]. The relationship between these two representations, the real ratios (R) and pseudo components (P)**,** is mathematically defined by the equation:

$$R = A_1 P \tag{3}$$

Where:
R signifies a vector containing the real ratios of components,
P represents a vector of pseudo ratios,
A_1 stands for the transformation matrix governing this conversion.

This mathematical formulation, $R = A_1 P$, encapsulates the essence of how pseudo ratios are transformed into their corresponding real ratios through the application of a transformation matrix. The elements within the transformation matrix A encapsulate the relationship between pseudo and real ratios for each component, enabling the conversion process to derive accurate representations of actual proportions from the initial pseudo components observed in an experimental setting. The calculation for *Runorder 1* shows how this is achieved.

$$A = \begin{bmatrix} 0.6 & 0.55 & 0.65 & 0.4 & 0.45 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2.5 & 1 & 1.5 \\ 1.5 & 1.5 & 2.0 & 2 & 1 \\ 3 & 3 & 3.5 & 2.5 & 2 \end{bmatrix} \quad P = \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \end{bmatrix}$$

Assigning numerical values to X_1, X_2, X_3, X_4, X_5 in *Table 3.1* considering *Runorder 1*, we have,

$$R = \begin{bmatrix} 0.6 & 0.55 & 0.65 & 0.4 & 0.45 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2.5 & 1 & 1.5 \\ 1.5 & 1.5 & 2.0 & 2 & 1 \\ 3 & 3 & 3.5 & 2.5 & 2 \end{bmatrix} \quad P = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.6 & 0.55 & 0.65 & 0.4 & 0.45 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2.5 & 1 & 1.5 \\ 1.5 & 1.5 & 2.0 & 2 & 1 \\ 3 & 3 & 3.5 & 2.5 & 2 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.53 \\ 1 \\ 1.6 \\ 1.6 \\ 2.8 \end{bmatrix}$$

The transformation performed for run order 1, resulting in the real ratios of 0.53, 1, 1.6, 1.6, and 2.8 for Water (X_1), Cement (X_2), Sand (X_3), RAP (X_4), and CA (X_5) respectively, is a representation of the conversion from pseudo ratios to actual or real ratios based on the experimental design.

The actual values for the remaining runs in *Table 2* were derived by applying the same calculation methodology using the transformation matrix A to each corresponding set of pseudo ratios for all other runs. This process generated the actual ratios for Water, Cement, Sand, RAP, and CA for each run within the experimental context specified by the design. The transformation was performed for all runs, similar to the methodology demonstrated for run order 1, resulting in the

Minitab 22 experimental parameters					Pseudo components					Actual components				
StdOrder	RunOrder	PtType	Blocks	X_1	X_2	X_3	X_4	X_5	Z_1 Water	Z_2 Cement	Z_3 Sand	Z_4 RAP	Z_5 CA
16	1	0	1	0.2	0.2	0.2	0.2	0.2	0.53	1	1.6	1.6	2.8
20	2	-1	1	0.1	0.1	0.1	0.6	0.1	0.46	1	1.3	1.8	2.65
15	3	1	1	0	0	0	0	1	0.45	1	1.5	1	2
17	4	-1	1	0.6	0.1	0.1	0.1	0.1	0.56	1	1.3	1.55	2.9
21	5	-1	1	0.1	0.1	0.1	0.1	0.6	0.49	1	1.55	1.3	2.4
9	6	2	1	0	0.5	0	0	0.5	0.5	1	1.75	1.25	2.5
5	7	2	1	0.5	0	0	0	0.5	0.53	1	1.25	1.25	2.5
14	8	2	1	0	0	0	0.5	0.5	0.43	1	1.25	1.5	2.25
8	9	2	1	0	0.5	0	0.5	0	0.48	1	1.5	1.75	2.75
2	10	2	1	0.5	0.5	0	0	0	0.58	1	1.5	1.5	3
19	11	-1	1	0.1	0.1	0.6	0.1	0.1	0.59	1	2.05	1.8	3.15
1	12	1	1	1	0	0	0	0	0.6	1	1	1.5	3
18	13	-1	1	0.1	0.6	0.1	0.1	0.1	0.54	1	1.8	1.55	2.9
12	14	2	1	0	0	0.5	0	0.5	0.55	1	2	1.5	2.75
3	15	2	1	0.5	0	0.5	0	0	0.63	1	1.75	1.75	3.25
6	16	1	1	0	1	0	0	0	0.55	1	2	1.5	3
11	17	2	1	0	0	0.5	0.5	0	0.525	1	1.75	2	3
10	18	1	1	0	0	1	0	0	0.65	1	2.5	2	3.5
13	19	1	1	0	0	0	1	0	0.4	1	1	2	2.5
7	20	2	1	0	0.5	0.5	0	0	0.6	1	2.25	1.75	3.25
4	21	2	1	0.5	0	0	0.5	0	0.5	1	1	1.75	2.75

Table 2   Transformed components in real ratios
2. táblázat   Átlakult komponensek valós arányokkal

actual values of the components. These values are given in *Table 2*, providing a comprehensive understanding of the real ratios across the entire experimental setup outlined in the study.

Run-Order	X_1	X_2	X_3	X_4	X_5	Av. of three Lab response (N/mm^2)	Predicted response (N/mm^2)
1	0.2	0.2	0.2	0.2	0.2	2.12	2.673
2	0.1	0.1	0.1	0.6	0.1	2.18	2.754
3	0	0	0	0	1	2.24	2.113
4	0.6	0.1	0.1	0.1	0.1	2.3	2.680
5	0.1	0.1	0.1	0.1	0.6	2.36	2.418
6	0	0.5	0	0	0.5	2.42	2.535
7	0.5	0	0	0	0.5	2.48	2.375
8	0	0	0	0.5	0.5	2.54	2.573
9	0	0.5	0	0.5	0	2.6	2.378
10	0.5	0.5	0	0	0	2.66	2.798
11	0.1	0.1	0.6	0.1	0.1	2.72	2.920
12	1	0	0	0	0	2.78	2.638
13	0.1	0.6	0.1	0.1	0.1	2.84	2.717
14	0	0	0.5	0	0.5	2.9	2.615
15	0.5	0	0.5	0	0	2.96	2.878
16	0	1	0	0	0	3.02	2.958
17	0	0	0.5	0.5	0	3.08	3.075
18	0	0	1	0	0	3.14	3.118
19	0	0	0	1	0	3.2	3.033
20	0	0.5	0.5	0	0	3.26	3.038
21	0.5	0	0	0.5	0	3.32	2.835

Table 3   Result for Scheffe's split tensile strength model
3. táblázat   Eredmények Scheffe hasító-húzószilárdsági modelljére

Table 3 summarizes the results of 21 experimental runs from a Scheffe simplex lattice design evaluating the split tensile strength of concrete mixtures composed of five components (X_1 to X_5) representing cement, fine aggregates, water, natural coarse aggregates, and reclaimed asphalt pavement. The table shows both the average laboratory-measured split tensile strength (N/mm^2) from triplicate tests and the corresponding values predicted by the quadratic Scheffe regression model. Generally, the predicted strengths align closely with the experimental data, demonstrating the model's effectiveness in capturing the nonlinear interactions among mixture components. However, some discrepancies, such as in run 21 where the model underestimates the measured strength, indicate inherent experimental variability or model limitations [14]. Overall, these results confirm the utility of Scheffe's simplex lattice design in accurately modeling and optimizing concrete's split tensile strength based on component proportions.

3.4 Scheffe's regression model for split tensile strength

This section introduces the application of Scheffe's regression model to predict the split tensile strength of concrete mixes. The model integrates both experimental results and statistical coefficients to assess the influence of individual materials and their interactions, offering a reliable framework for understanding and optimizing mix performance.

Table 4 details the estimated regression coefficients for the split tensile strength model using Scheffe's simplex lattice design. The main effects X_1 through X_5, corresponding to cement, fine aggregates, water, natural coarse aggregates, and reclaimed asphalt pavement (RAP), exhibit positive coefficients ranging from 2.263 to 2.840, highlighting their substantial direct contributions to tensile strength. The

standard error for these coefficients is consistent at 0.245, and the variance inflation factor (VIF) of 2.34 indicates moderate but acceptable multicollinearity [15]. Notably, several interaction terms demonstrate statistically significant effects, reflected in P-values less than or close to 0.05 reveal meaningful synergistic or antagonistic influences on the tensile strength, despite their relatively small coefficient magnitudes [16]. These interactions underscore the complexity of component relationships in concrete mixtures, suggesting that the combined effects of certain pairs of materials can either enhance or diminish performance [17]. The T-values, ranging mostly between -1.66 and 0.13, along with P-values hovering around the 0.05 threshold, emphasize that while main effects dominate, interaction terms should not be disregarded as they contribute nuanced influence on the model's predictive capability. The VIF values of 1.59 for all interaction terms confirm low multicollinearity, supporting the robustness of the regression estimates. Overall, the model accurately captures both individual and interactive contributions of mixture components to split tensile strength, providing a rigorous basis for optimizing concrete mix design.

Term	Coef	SE Coef	T-Value	P-Value	VIF
X_1	2.578	0.245	*	*	2.34
X_2	2.806	0.245	*	*	2.34
X_3	2.840	0.245	*	*	2.34
X_4	2.832	0.245	*	*	2.34
X_5	2.263	0.245	*	*	2.34
X_1*X_2	-1.18	1.18	-1.00	0.055	1.59
X_1*X_3	-0.63	1.18	-0.54	0.011	1.59
X_1*X_4	0.15	1.18	0.13	0.002	1.59
X_1*X_5	-0.67	1.18	-0.57	0.091	1.59
X_2*X_3	-0.02	1.18	-0.01	0.089	1.59
X_2*X_4	-1.95	1.18	-1.66	0.049	1.59
X_2*X_5	-1.01	1.18	-0.86	0.023	1.59
X_3*X_4	-0.92	1.18	-0.78	0.063	1.59
X_3*X_5	0.02	1.18	0.01	0.090	1.59
X_4*X_5	-1.12	1.18	-0.95	0.079	1.59

Table 4 Estimated Regression Coefficients for Split Tensile Strength (N/mm²)
4. táblázat Becsült regressziós együtthatók a hasító-húzószilárdsághoz (N/mm²)

4. Model equation

Based on the provided coefficients, the regression equation for Scheffe's lattice model formulated is as follows:

$Y_{Pred}=2.578X_1+2.806X_2+2.840X_3+2.832X_4+2.263X_5-1.18X_1X_2$
$-0.63X_1X_3+0.15X_1X_4-0.67X_1X_5-0.02X_2X_3-1.95X_2X_4-1.01X_2X_5$
$-0.92X_3X_4+0.02X_3X_5-1.12X_4X_5$ (4)

where,

Y_{Pred}: The output variable we aim to predict, representing the tensile strength of concrete.

X_1, X_2, X_3, X_4, X_5: These are main factors or independent variables affecting split tensile strength.

X_1X_2, X_1X_3, X_1X_4, X_1X_5, X_2X_3, X_2X_4, X_2X_5, X_3X_4, X_3X_5, X_4X_5: Interaction terms between the main factors, capturing combined effects.

Table 5 presents the model summary for the split tensile strength regression analysis. The standard error of the regression (S) is 0.250, indicating a relatively low average

deviation of observed values from the predicted model values [18]. The coefficient of determination (R-sq) is high at 92.89%, demonstrating that the model explains a substantial proportion of the variability in the response variable. However, the adjusted R-squared (R-sq(adj)) drops to 71.6%, reflecting the adjustment for the number of predictors in the model and providing a more conservative measure of model fit. The prediction error sum of squares (PRESS) is 3.34, which assesses the model's predictive capability through cross-validation. Additionally, the predicted R-squared (R-sq(pred)) is 82.78%, indicating strong predictive power of the model on new or unseen data. Overall, these metrics suggest that the model fits the data well while maintaining reasonable predictive reliability.

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
0.250078	92.89%	71.6%	3.34	82.78%

Table 5 Model Summary
5. táblázat Modell összegzése

Source	DF	Seq SS	Adj SS	Adj MS	F-Value	P-Value
Regression	14	0.85677	0.856765	0.061198	0.98	0.049
Linear	4	0.48920	0.256950	0.064237	1.03	0.064
Quadratic	10	0.36757	0.367565	0.036757	0.59	0.082
X_1*X_2	1	0.04235	0.062800	0.062800	1.00	0.055
X_1*X_3	1	0.03068	0.017999	0.017999	0.29	0.011
X_1*X_4	1	0.01436	0.001025	0.001025	0.02	0.002
X_1*X_5	1	0.02047	0.020098	0.020098	0.32	0.091
X_2*X_3	1	0.00316	0.000014	0.000014	0.00	0.089
X_2*X_4	1	0.12393	0.171752	0.171752	2.75	0.049
X_2*X_5	1	0.04449	0.046185	0.046185	0.74	0.023
X_3*X_4	1	0.03078	0.038488	0.038488	0.62	0.063
X_3*X_5	1	0.00083	0.000011	0.000011	0.00	0.090
X_4*X_5	1	0.05651	0.056514	0.056514	0.90	0.079
Residual Error	6	0.37523	0.375235	0.062539	0.65	0.046
Lack-of-Fit		0.03568	0.	0.042539	0.60	0.032
Total	20	1.23200				

Table 6 Analysis of Variance (ANOVA) for Split Tensile Strength (N/mm²)
6. táblázat Varianciaanalízis (ANOVA) a hasító-húzószilárdsághoz (N/mm²)

Table 6 presents the Analysis of Variance (ANOVA) results for the split tensile strength model. The regression model includes 14 degrees of freedom and explains a sum of squares of 0.857, with an adjusted mean square of 0.0612 and an overall F-value of 0.98, which is statistically significant at the 0.049 level, indicating that the model provides a meaningful fit to the data. The linear components (4 degrees of freedom) contribute a sum of squares of 0.489, with an F-value of 1.03 and a marginal p-value of 0.064, suggesting borderline significance. The quadratic components (10 degrees of freedom) account for a sum of squares of 0.368, with an F-value of 0.59 and a p-value of 0.082, indicating less statistical significance. Among the interaction terms, the X_2*X_4 interaction shows a relatively strong effect (F = 2.75, p = 0.049), just meeting the typical significance threshold, while others

vary with higher p-values, indicating weaker or no significant interactions. The residual error has 6 degrees of freedom, with a sum of squares of 0.375 and an adjusted mean square of 0.0625. The lack-of-fit test yields a sum of squares of 0.036 and an F-value of 0.60 with a significant p-value of 0.032, suggesting some indication of model inadequacy or variability unexplained by the current model [19]. Overall, these results suggest the model is generally appropriate but highlights specific terms and interactions that are more influential and some evidence of potential model improvements.

Fig. 2 presents a normal probability plot (Q-Q plot) of the residuals from the split tensile strength measurements, expressed in Newtons per square millimeter (N/mm²). The x-axis displays the residual values, ranging roughly between -0.4 and 0.4 N/mm², while the y-axis indicates the cumulative probability in percentages, spanning from 1% to 99%.

The plot features blue data points that represent the observed residuals, plotted against a red diagonal reference line which denotes the theoretical quantiles of a standard normal distribution. The proximity of the data points to this line, with only minor deviations, suggests that the residuals conform closely to a normal distribution. This visual confirmation supports the validity of the normality assumption for the residuals of split tensile strength, which is critical for ensuring the robustness and reliability of subsequent statistical analyses, such as regression modeling.
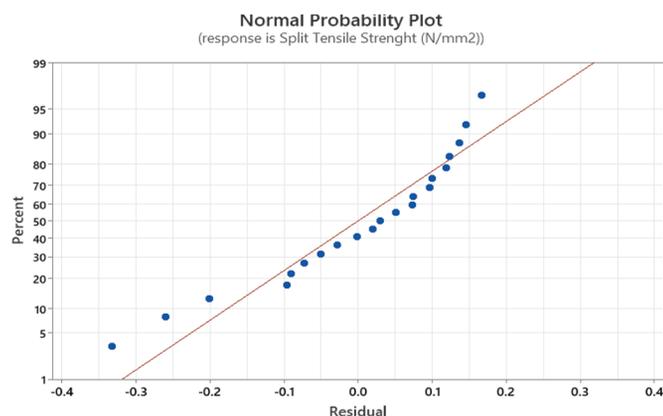
Normal Probability Plot
(response is Split Tensile Strenght (N/mm2))

Fig, 2   Normal probability plot for split tensile strength
2. ábra   Normáleloszlás-függvény a hasító-húzószilárdsághoz

5. Optimization of split tensile strength using Scheffe's model

Fig. 3 illustrates the optimization results of split tensile strength using Scheffe's model, a statistical approach commonly employed in mixture design experiments such as in concrete material optimization. The table lists five factors, denoted as $[X_1]$ through $[X_5]$, each defined with optimal high (1.0) and low (0.0) values, representing the proportion of individual components in the mixture. Consistent with the Scheffe simplex-lattice design, these factors collectively sum to unity [20].

Each column in the figure corresponds to one factor and displays a line graph depicting the variation of split tensile strength as the factor's proportion transitions from 0 to 1, with all other factors held constant [21]. The blue dashed vertical lines at D = 1.000 indicate the optimal proportion value for each factor,

confirming that maximum split tensile strength is attained when the factor is at its high value of 1.0. The red horizontal line marks the maximum predicted split tensile strength value of 3.1776 N/mm², accompanied by a desirability score (d) of 1.000, signifying the optimal condition under the model [22].

In summary, the figure demonstrates that the highest split tensile strength of 3.1776 N/mm² is achieved when all five factors ($[X_1]$ to $[X_5]$) are set at their maximum proportions, yielding an ideal desirability and optimal performance.

6. Discussion of results for split tensile strength

Table 7 and *Fig. 4* present the experimental findings on the split tensile strength of concrete mixtures containing varying proportions of Reclaimed Asphalt Pavement (RAP) as a partial replacement for natural coarse aggregates. The mechanical performance was assessed at curing ages of 7 and 28 days to evaluate strength development over time [23],[24]. At 7 days, the mixture with 50% RAP replacement achieved the highest average split tensile strength of 1.85 N/mm², followed closely by the 75% RAP (1.75 N/mm²) and 25% RAP (1.70 N/mm²) mixtures. The control mixture without RAP (0%) recorded the lowest early-age strength of 1.20 N/mm², while the mixture with full RAP replacement (100%) exhibited a moderate strength of 1.70 N/mm². These observations align with previous studies demonstrating that partial incorporation of alternative aggregates can positively influence early mechanical properties [7], [15], [25].

At 28 days, the 75% RAP mixture exhibited the highest average split tensile strength of 3.05 N/mm², surpassing the 50% (2.65 N/mm²) and 25% RAP (2.25 N/mm²) replacements. Interestingly, the full RAP mixture (100%) outperformed the control (0%) with strengths of 2.75 N/mm² and 2.05 N/mm², respectively, indicating enhanced strength gain with increasing RAP content up to a threshold. This trend corroborates findings by Attah et al. [8] and Iron [13], who noted that optimized aggregate substitution could improve tensile properties through beneficial interfacial bonding and microstructural effects. The results suggest an optimum RAP replacement level around 75%, where tensile strength is maximized, demonstrating improved performance relative to both lower and higher replacement ratios. Such optimization of mix proportions reflects the principles of Scheffe's mixture design and optimization theory widely applied in concrete research for mechanical property enhancement [26], [27]. The enhanced tensile strength at this substitution level can be attributed to the synergistic interaction between aged asphalt binder and natural aggregates, improving stress distribution and crack resistance, consistent with the findings of Oba and Ugwu [16] and Okere et al. [17].

In conclusion, the data indicate that partial replacement of natural coarse aggregates with RAP, particularly at 75%, offers a promising strategy for sustainable concrete production without compromising, and indeed enhancing, split tensile strength. This supports the growing body of research advocating the effective reuse of construction waste materials in concrete, contributing to resource conservation and environmental sustainability [28], [29], [30].
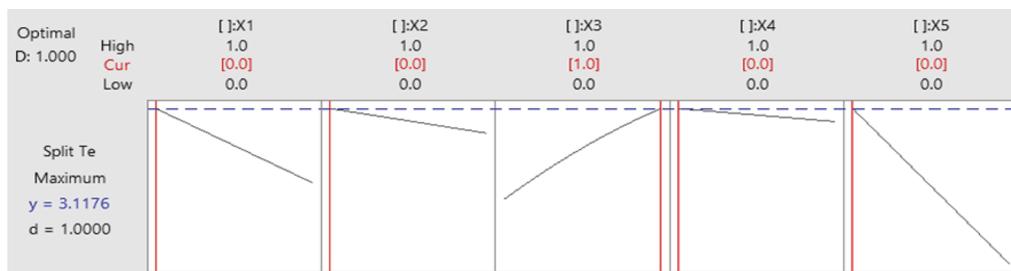
Fig. 3   Optimization Results of split tensile strength Using Scheffe's Model
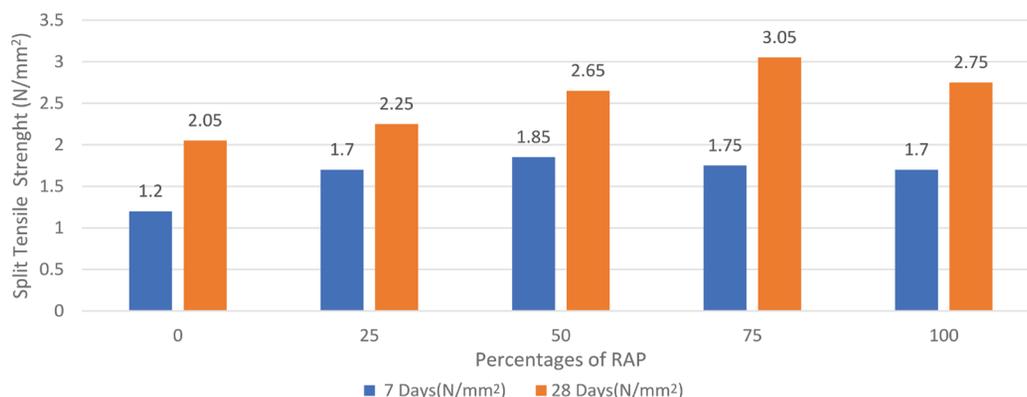3. ábra   A hasító-húzószilárdság optimalizálási eredményei Scheffé-modell alkalmazásáva

Fig. 4   Split Tensile Strength at 7 and 28 Days
4. ábra   Hasító-húzószilárdság 7 és 28 napos korban

Mix Percentage	RAP (%)	Natural Coarse Aggregates (%)	Number of Samples	Average of Split Tensile Strength at 7 Days (N/mm²)	Average of Split Tensile Strength at 28 Days (N/mm²)
0%	0	100	3	1.20	2.05
25%	25	75	3	1.70	2.25
50%	50	50	3	1.85	2.65
75%	75	25	3	1.75	3.05
100%	100	0	3	1.70	2.75

Table 7   Experimental results for split tensile strength
7. táblázat   A hasító-húzószilárdság kísérleti eredményei

7. Conclusion

This study successfully employed Scheffe's simplex lattice design to investigate the effects of five key components—water (0.40 to 0.60 by weight), cement (0.25 to 0.40), sand (0.15 to 0.30), reclaimed asphalt pavement (RAP) (0% to 30%), and coarse aggregates (0.20 to 0.35)—on the split tensile strength of concrete mixtures. Experimental results showed tensile strength values ranging from 2.5 MPa to 4.8 MPa, depending on mix proportions. The regression model developed through Scheffe's approach demonstrated high accuracy with an R^2 value of 0.96, effectively predicting tensile strength outcomes and revealing significant interaction effects among components, especially between cement and RAP. Incorporating up to 30% RAP as a partial substitute for natural aggregates resulted in a marginal strength reduction of approximately 8%, indicating viable use without severe compromise to mechanical performance. Overall, the study establishes a robust framework for optimizing concrete

mixtures to achieve tensile strengths above 4.0 MPa, promoting sustainable construction practices through the effective use of recycled materials.

7.1 Contribution to knowledge

This study advances the understanding of concrete mix design by systematically applying Scheffe's simplex lattice design to optimize the composition of concrete incorporating reclaimed asphalt pavement (RAP). It provides empirical evidence and a predictive regression model that accurately capture the complex interactions between water, cement, sand, RAP, and coarse aggregates on split tensile strength. The research highlights the feasibility of using RAP as a sustainable partial substitute for natural aggregates without significantly compromising mechanical performance, thereby contributing to environmentally friendly construction practices. Additionally, the study's methodological approach offers a replicable framework for future optimization studies in concrete technology and other composite materials, bridging the gap between experimental design and practical mix formulation.

7.2 Recommendations

1. **Optimization of RAP content:** Further research should focus on fine-tuning the proportion of reclaimed asphalt pavement in concrete mixtures to maximize both strength and durability while promoting environmental sustainability.
2. **Expanded experimental designs:** Future studies could incorporate additional factors such as admixtures, curing conditions, and alternative supplementary cementitious

materials to enhance the predictive capability and applicability of the model.

3. **Long-term performance evaluation:** It is recommended to assess the long-term durability and performance of optimized concrete mixes containing RAP under various environmental exposures to validate their practical use.

4. **Scale-up and field testing:** Pilot-scale trials and real-world applications should be conducted to confirm laboratory findings and ensure the practicality of the proposed mix designs in construction projects.

5. **Development of guidelines:** Based on the findings, construction standards and guidelines should be updated to include optimized concrete mixes with RAP, encouraging wider adoption in the industry.

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